Online Supplemental Material

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- Section A: Bayesian estimation steps and full conditional distributions for model-based imputation
- Section B: Description of model-based imputation for 3-level computer simulation model
- Section C: Trellis plots displaying relative bias and confidence interval coverage from all conditions of the computer simulation.

A. MCMC Sampling Steps and Distributions for Two-Level Imputation

This document gives technical details of the two-level Gibbs sampler, specifically the full conditional distributions used to draw model parameters, random effects, latent means, and missing values for model-based imputation in Blimp.

Gibbs Sampler Steps for the Analysis Model

In this section we abandon the scalar notation from the manuscript in favor of a more succinct matrix representation of the multilevel model

$$\boldsymbol{y}_j = \mathbf{X}_j\boldsymbol{\beta} + \mathbf{Z}_j\boldsymbol{b}_j + \boldsymbol{\varepsilon}_j \tag{SA1}$$

where $\boldsymbol{y}_j$ is the vector of outcome scores for cluster j, $\mathbf{X}_j$ is the corresponding matrix of predictor variables (level-1 or level-2), including a unit vector for the intercept, $\mathbf{Z}_j$ is a subset of the level-1 variables in $\mathbf{X}_j$ that have a random influence on the outcome (e.g., a unit vector and any random coefficient predictors), $\boldsymbol{b}_j$ is the column vector of level-2 residuals for cluster j, and $\boldsymbol{\varepsilon}_j$ is a vector of within-cluster residuals. A variety of sources give the full conditional distributions for this model (Browne, 1998; Browne & Draper, 2000; Enders et al., 2018; Lynch, 2007; Schafer & Yucel, 2002; Yucel, 2008), which we summarize here for completeness.

To illustrate the following steps more concretely, we will refer to the following substantive model, which includes within-cluster and cross-level interaction effects. Between-cluster interactions involving pairs of level-2 variables are also possible, but it should become evident that the composition of the analysis model has no bearing on the covariate models.

$$\begin{aligned}y_{ij} = \beta_0 + \beta_1(x_{1ij}) + \beta_2(x_{2ij}) + \beta_3(x_{3ij}) + \beta_4(x_{1ij})(x_{2ij}) + \beta_5(x_{4j}) \\ + \beta_6(x_{5j}) + \beta_7(x_{3ij})(x_{5j}) + b_{0j} + b_{1j}(x_{1ij}) + \varepsilon_{ij} \\ \binom{b_{0j}}{b_{1j}} \sim MN(0, \boldsymbol{\Sigma}_b) \quad \varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)\end{aligned} \tag{SA2}$$

Step 1: Draw regression coefficients from $p(\boldsymbol{\beta}|\cdot) \propto p(Y|\boldsymbol{\beta}, \boldsymbol{b}_j, \boldsymbol{\Sigma}_b, \sigma_\varepsilon^2, X)p(\boldsymbol{\beta})$. Assuming a uniform prior, $p(\boldsymbol{\beta}) \propto 1$, the full conditional distribution is a multivariate normal distribution.

$$\begin{aligned}\boldsymbol{\beta} \sim MN\left(\hat{\boldsymbol{\beta}}, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\beta}}}\right) \\ \hat{\boldsymbol{\beta}} = \left(\sum_{j=1}^{J} \mathbf{X}_j'\mathbf{X}_j\right)^{-1} \sum_{j=1}^{J} \mathbf{X}_j'(\boldsymbol{y}_j - \mathbf{Z}_j\boldsymbol{b}_j) \\ \boldsymbol{\Sigma}_{\hat{\boldsymbol{\beta}}} = \sigma_\varepsilon^2 \left(\sum_{j=1}^{J} \mathbf{X}_j'\mathbf{X}_j\right)^{-1}\end{aligned} \tag{SA3}$$

Applied to the model from Equation SA2, the **X** matrix includes a unit vector for the intercept and a column for each explanatory variable and interaction term, the **Z** matrix is comprised of a unit vector and X_1.

Step 2: Draw random effects for cluster j from a multivariate normal distribution.

$$\begin{aligned}\boldsymbol{b}_j \sim MN\left(\hat{\boldsymbol{b}}_j, \mathbf{V}_{\boldsymbol{b}_j}\right) \\ \mathbf{V}_{\boldsymbol{b}_j} = (\sigma_\varepsilon^{-2}\ \mathbf{Z}_j'\mathbf{Z}_j + \boldsymbol{\Sigma}_b^{-1})^{-1} \\ \hat{\boldsymbol{b}}_j = \sigma_\varepsilon^{-2}\ \mathbf{V}_{\boldsymbol{b}_j}\mathbf{Z}_j'(\boldsymbol{y}_j - \mathbf{X}_j\boldsymbol{\beta})\end{aligned} \tag{SA4}$$

Step 3: Draw the residual variance from $p(\sigma_\varepsilon^2|\cdot) \propto p(Y|\boldsymbol{\beta}, \boldsymbol{b}_j, \boldsymbol{\Sigma}_b, \sigma_\varepsilon^2, X)p(\sigma_\varepsilon^2)$. We define $1/\sigma_\varepsilon^2$ as a gamma random variable and draw the reciprocal of the residual variance (i.e., the precision) from a gamma distribution

$$1/\sigma_\varepsilon^2 \sim \mathrm{G}\left(\frac{N + df_p}{2}, \frac{S + S_p}{2}\right)$$
$$S = \sum_{j=1}^{J} \boldsymbol{\varepsilon}_j' \boldsymbol{\varepsilon}_j \tag{SA5}$$
$$\boldsymbol{\varepsilon}_j = \boldsymbol{y}_j - \mathbf{X}_j\boldsymbol{\beta} - \mathbf{Z}_j\boldsymbol{b}_j$$

with hyperparameters df_p and S_p for the prior distribution. The default setting in Blimp specifies $S_p = 1$ and $df_p = 2$, which corresponds to a gamma(1,.5) prior. Two other options are to set $S_p = 0$ and $df_p = -2$ (the PRIOR2 keyword of the OPTIONS command) and $S_p = 0$ and $df_p = 0$ (the PRIOR3 keyword), a Jeffreys prior.

Step 4: Draw the between-cluster covariance matrix variance from $p(\boldsymbol{\Sigma}_b|\cdot) \propto p(Y|\boldsymbol{\beta}, \boldsymbol{b}_j, \boldsymbol{\Sigma}_b, \sigma_\varepsilon^2, X)p(\boldsymbol{\Sigma}_b)$. We define the inverse of the covariance matrix (i.e., the precision matrix, $\boldsymbol{\Sigma}_b^{-1}$) as a Wishart random variable. The level-2 precision matrix is sampled from a Wishart distribution, conditional on the current parameter estimates, level-2 residuals, and imputations.

$$\boldsymbol{\Sigma}_b^{-1} \sim \mathrm{W}((\mathbf{S} + \mathbf{S}_p^{-1})^{-1}, J + df_p) \tag{SA6}$$
$$\mathbf{S} = \sum_{j=1}^{J} \boldsymbol{b}_j' \boldsymbol{b}_j$$

$\mathbf{S}_p$ can be viewed as the *inverse* of the prior sums of squares matrix based on df_p degrees of freedom (i.e., prior observations). As such, $\mathbf{S} + \mathbf{S}_p^{-1}$ is a sums of squares and cross products matrix based on $J + df_p$ observations. The default prior sets $\mathbf{S}_p^{-1}$ $= \mathbf{I}$ and $df_p = p + 1$, where p is the dimension of $\boldsymbol{\Sigma}_b$. This prior corresponds to marginal uniform priors between -1 and 1 for all correlations and a marginal inverse gamma prior IG(1,.5) for variance elements. Specifying the PRIOR2 keyword of the OPTIONS command sets $\mathbf{S}_p^{-1} = 0$ and $df_p = -p - 1$, which is equivalent to a uniform prior on the elements in $\boldsymbol{\Sigma}_b$. Finally, the PRIOR3 keyword sets $\mathbf{S}_p^{-1} = 0$ and $df_p = 0$. For random intercept models with a single level-2 variance component, we draw the reciprocal of the variance, $1/\sigma_b^2$, from a gamma random variable with analogous univariate priors based on $p = 1$.

Step 5: Draw the imputation for observation i in cluster j from a univariate normal posterior distribution.

$$y_{ij(mis)} = N(\boldsymbol{x}_{ij}\boldsymbol{\beta} + \boldsymbol{z}_{ij}\boldsymbol{b}_j, \sigma_\varepsilon^2) \tag{SA7}$$

Gibbs Sampler Steps for the Covariate Model *r*

To convey the estimation steps in the most general way possible, we introduce new notation that differs from that in the manuscript. To begin, index the P level-1 predictors as $p = 1$, ..., P, and index the Q level-2 predictors as $q = 1$, ..., Q. As explained in the paper, each level-1 variable has a regression model for the observations and a regression model for its latent cluster means. Thus, level-1 estimation involves P computational cycles, and level-2 estimation requires $R = P + Q$ computational cycles. To simplify the notation, we index the entire set of variables as $r = 1$, ..., R, such that $r \leq P$ corresponds to either a level-1 observation or its corresponding level-2 group mean, and $r > P$ refers to a manifest level-2 variable.

Using generic notation, the level-1 and level-2 regression models are given below.

$$\begin{gathered} x_{r,ij} = \mu_{r,j} + \tilde{\boldsymbol{x}}_{-r,ij}\boldsymbol{\gamma}_r + e_{r,ij} \\ e_{r,ij} \sim N(0, \sigma^2_{e_r}) \end{gathered} \tag{SA8}$$

$$\begin{gathered} x_{r,j} = \mu_r + \tilde{\boldsymbol{x}}_{-r,j}\boldsymbol{\eta}_r + \zeta_{r,j} \\ \zeta_{r,j} \sim N(0, \sigma^2_{\zeta_r}) \end{gathered} \tag{SA9}$$

where $x_{r,ij}$ is the level-1 score for covariate r, $\tilde{\boldsymbol{x}}_{-r,ij}$ denotes the $P - 1$ row vector of all other level-1 predictor variables except r, centered at their latent group means. Turning to the between-cluster regression, the outcome, $x_{r,j}$, is either a latent group mean (e.g., $x_{r,j} = \mu_{r,j}$) when $r \leq P$ or a manifest level-2 variable when $r > P$, and $\tilde{\boldsymbol{x}}_{-r,j}$ is a $R - 1$ row vector of grand-mean centered level-2 variables other than r. For some of the Gibbs sampling steps, it is convenient to concatenate observation-level quantities into matrices. For example, the N-row vector of level-1 outcome scores is $\boldsymbol{x}_{r,ij}$ and the corresponding N by $P - 1$ matrix of centered level-1 predictors is $\widetilde{\boldsymbol{X}}_{-r,ij}$. Similarly, the J-row vector of level-2 outcome scores is $\boldsymbol{x}_{r,j}$ and the J by R matrix of mean-centered predictors is $\widetilde{\boldsymbol{X}}_{-r,j}$.

To make the notation more concrete, consider the analysis model from Equation SA2. The multivariate normality assumption induces the following level-1 regression models.

$$\begin{aligned} x_{1ij} &= \mu_{1j} + \gamma_{11}(x_{2ij} - \mu_{2j}) + \gamma_{12}(x_{3ij} - \mu_{3j}) + e_{1ij} \\ x_{2ij} &= \mu_{2j} + \gamma_{21}(x_{1ij} - \mu_{1j}) + \gamma_{22}(x_{3ij} - \mu_{3j}) + e_{2ij} \\ x_{3ij} &= \mu_{3j} + \gamma_{31}(x_{1ij} - \mu_{1j}) + \gamma_{32}(x_{2ij} - \mu_{2j}) + e_{3ij} \end{aligned} \tag{SA10}$$

For any given model, $\boldsymbol{x}_{r,ij}$ is N-row vector of outcome scores, $\boldsymbol{\gamma}_r$ is a 2-element vector of level-1 regression coefficients, and $\widetilde{\boldsymbol{X}}_{-r,ij}$ is the N by 2 matrix of latent group mean centered predictors on the right side of each equation. The multivariate normality assumption also induces the following level-2 regression models.

$$\begin{aligned}
\mu_{1j} &= \mu_1 + \eta_{11}(\mu_{2j} - \mu_2) + \eta_{12}(\mu_{3j} - \mu_3) + \eta_{13}(x_{4j} - \mu_4) + \eta_{14}(x_{5j} - \mu_5) + \zeta_{1j} \\
\mu_{2j} &= \mu_2 + \eta_{21}(\mu_{1j} - \mu_1) + \eta_{22}(\mu_{3j} - \mu_3) + \eta_{23}(x_{4j} - \mu_4) + \eta_{24}(x_{5j} - \mu_5) + \zeta_{2j} \\
\mu_{3j} &= \mu_3 + \eta_{31}(\mu_{1j} - \mu_1) + \eta_{32}(\mu_{2j} - \mu_2) + \eta_{33}(x_{4j} - \mu_4) + \eta_{34}(x_{5j} - \mu_5) + \zeta_{3j} \\
x_{4j} &= \mu_4 + \eta_{41}(\mu_{1j} - \mu_1) + \eta_{42}(\mu_{2j} - \mu_2) + \eta_{43}(x_{3j} - \mu_3) + \eta_{44}(x_{5j} - \mu_5) + \zeta_{4j} \\
x_{5j} &= \mu_5 + \eta_{51}(\mu_{1j} - \mu_1) + \eta_{52}(\mu_{2j} - \mu_2) + \eta_{53}(x_{3j} - \mu_3) + \eta_{54}(x_{4j} - \mu_4) + \zeta_{5j}
\end{aligned} \tag{SA11}$$

For any given model, $\boldsymbol{x}_{r,j}$ is the J-row vector of outcome scores (latent means or manifest variables), $\boldsymbol{\eta}_r$ is a 4-element vector of level-2 regression coefficients, and $\widetilde{\boldsymbol{X}}_{-r,j}$ is the J by 4 matrix of grand mean centered predictors on the right side of each equation.

Step 6. If variable r is measured at level-1 (i.e., $r \leq P$), draw its latent cluster means from $p(\mu_{r,j}|\cdot) \propto p(X_r|\mu_r, \mu_{r,j}, \boldsymbol{\gamma}_r, \sigma^2_{e_r}, X_{-r})p(\mu_{r,j}|, \boldsymbol{x}_{-r,j}, \boldsymbol{\eta}_r, \sigma^2_{\zeta_r})$, which is the univariate normal distribution below.

$$p(\mu_{r,j}|\cdot) = N\left(\frac{\sigma^2_{\zeta_r}\sum_{i=1}^{n_j}(x_{r,ij} - \tilde{\boldsymbol{x}}_{-r,ij}\boldsymbol{\gamma}_r) + \sigma^2_{e_r}(\mu_r + \tilde{\boldsymbol{x}}_{-r,j}\boldsymbol{\eta}_r)}{\sigma^2_{e_r} + n_j\sigma^2_{\zeta_r}}, \frac{\sigma^2_{e_r}\sigma^2_{\zeta_r}}{\sigma^2_{e_r} + n_j\sigma^2_{\zeta_r}}\right) \tag{SA12}$$

Note that $\sigma^2_{e_r}$ and $\sigma^2_{\zeta_r}$ residual variance for covariate r's observations and latent means, respectively.

Step 7: Draw the grand mean of variable r from $p(\mu_r | \cdot) \propto p\left(X_r | \mu_r, \boldsymbol{\eta}_r, \sigma^2_{\zeta_r}, \mu_{r,j}, \boldsymbol{\gamma}_r, \sigma^2_{e_r}, X_{-r}\right) p(\mu_r)$. Assuming a uniform prior, $p(\mu_r) \propto 1$, the full conditional distribution is a univariate normal distribution.

$$p(\mu_r | \cdot) = N\left(\frac{\sum_{j=1}^{J}(x_{r,j} - \tilde{\boldsymbol{x}}_{-r,j}\boldsymbol{\eta}_r)}{J}, \frac{\sigma^2_{\zeta_r}}{J}\right) \quad \text{(SA13)}$$

where J the number of level-2 units. As noted previously, $x_{r,j}$ is a latent cluster mean when variable r is measured at level-1, and it is a score when r is at level-2.

Step 8: If variable r is measured at level-1 (i.e., $r \leq P$), draw its within-cluster regression slopes from $p(\boldsymbol{\gamma}_r | \cdot) \propto p\left(X_r | \mu_r, \boldsymbol{\eta}_r, \sigma^2_{\zeta_r}, \mu_{r,j}, \boldsymbol{\gamma}_r, \sigma^2_{e_r}, X_{-r}\right) p(\boldsymbol{\gamma}_r)$. Because latent group means replace the regression intercept, the level-1 regression requires that the outcome (i.e., a level-1 score) is centered at its current group mean from step 9. In line with our previous notation, we denote the N-row vector of centered outcome scores as $\tilde{\boldsymbol{x}}_{r,ij}$. Assuming independent uniform priors, $p(\boldsymbol{\gamma}_r) \propto 1$, the full conditional distribution is a multivariate normal distribution.

$$p(\boldsymbol{\gamma}_r | \cdot) = MN(\hat{\boldsymbol{\gamma}}_r, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\gamma}}_r}) \quad \text{(SA14)}$$

$$\hat{\boldsymbol{\gamma}}_r = \left(\widetilde{\boldsymbol{X}}'_{-r,ij}\widetilde{\boldsymbol{X}}_{-r,ij}\right)^{-1}\widetilde{\boldsymbol{X}}'_{-r,ij}\tilde{\boldsymbol{x}}_{r,ij} \quad \text{(SA15)}$$

$$\boldsymbol{\Sigma}_{\hat{\boldsymbol{\gamma}}_r} = \sigma^2_{e_r}\left(\widetilde{\boldsymbol{X}}'_{-r,ij}\widetilde{\boldsymbol{X}}_{-r,ij}\right)^{-1} \quad \text{(SA16)}$$

Note that it is not necessary to account for clustering here because all scores are centered at their latent group means.

Step 9: If variable r is measured at level-1 (i.e., $r \leq P$), draw the within-cluster residual variance from $p(\sigma^2_{e_r}|\cdot) \propto p\big(X_r|\mu_r, \boldsymbol{\eta}_r, \sigma^2_{\zeta_r}, \mu_{r,j}, \boldsymbol{\gamma}_r, \sigma^2_{e_r}, X_{-r}\big)p(\sigma^2_{e_r})$. First, define a level-1 residual as follows

$$\hat{e}_{r,ij} = x_{r,ij} - \mu_{r,j} - \tilde{\boldsymbol{x}}_{-r,ij}\boldsymbol{\gamma}_r \tag{SA17}$$

and stack these residuals into an N-row vector define a N-row vector $\hat{\boldsymbol{e}}_{r,ij}$. We define $1/\sigma^2_e$ as a gamma random variable and draw the reciprocal of the residual variance (i.e., precision) from a gamma distribution

$$1/\sigma^2_e \sim \mathrm{G}\left(\frac{N + df_p}{2}, \frac{\hat{\boldsymbol{e}}'_{r,j}\hat{\boldsymbol{e}}_{r,j} + S_p}{2}\right) \tag{SA18}$$

with hyperparameters df_p and S_p for the prior distribution. The default setting in Blimp (XPRIOR1) specifies $S_p = 1$ and $df_p = 2$, which corresponds to a gamma(1,.5) prior. Two other options are to set $S_p = 0$ and $df_p = -2$ (the XPRIOR2 keyword of the OPTIONS command) and $S_p = 0$ and $df_p = 0$ (the XPRIOR3 keyword), a Jeffreys prior. Our simulation used a Jeffreys prior, but we have found that the default setting prevents between-cluster variances from collapsing when covariates have very low intraclass correlations.

Step 10: Draw between-cluster regression slopes for covariate r from $p(\boldsymbol{\eta}_r|\cdot) \propto p\big(X_r|\mu_r, \boldsymbol{\eta}_r, \sigma^2_{\zeta_r}, \mu_{r,j}, \boldsymbol{\gamma}_r, \sigma^2_{e_r}, X_{-r}\big)p(\boldsymbol{\eta}_r)$. Because the grand mean replaces the fixed regression intercept, the level-2 regression requires the outcome (i.e., a latent group mean or level-2 score) to be centered at its grand mean from step 6. In line with our previous notation, we denote the J-row vector of centered outcome scores as $\tilde{\boldsymbol{x}}_{r,j}$.

Assuming independent uniform priors, $p(\boldsymbol{\eta}_r) \propto 1$, the full conditional distribution is a multivariate normal distribution

$$p(\boldsymbol{\eta}_r | \cdot) = MN(\hat{\boldsymbol{\eta}}_r, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\eta}}_r}) \tag{SA19}$$

$$\hat{\boldsymbol{\eta}}_r = \left(\widetilde{\boldsymbol{X}}'_{-r,j}\widetilde{\boldsymbol{X}}_{-r,j}\right)^{-1}\widetilde{\boldsymbol{X}}'_{-r,j}\tilde{\boldsymbol{x}}_{r,j} \tag{SA20}$$

$$\boldsymbol{\Sigma}_{\hat{\boldsymbol{\eta}}_r} = \sigma^2_{\zeta_r}\left(\widetilde{\boldsymbol{X}}'_{-r,j}\widetilde{\boldsymbol{X}}_{-r,j}\right)^{-1} \tag{SA21}$$

Step 11: Draw the between-cluster residual variance for covariate r from $p(\sigma^2_{\zeta_r} | \cdot) \propto p(X_r | \mu_r, \boldsymbol{\eta}_r, \sigma^2_{\zeta_r}, \mu_{r,j}, \boldsymbol{\gamma}_r, \sigma^2_{e_r}, X_{-r})p(\sigma^2_{\zeta_r})$. First, define a J-row vector of level-2 residuals as

$$\hat{\boldsymbol{\zeta}}_{r,j} = \boldsymbol{x}_{r,j} - \boldsymbol{1}\mu_r - \widetilde{\boldsymbol{X}}_{-r,j}\boldsymbol{\eta}_r \tag{SA22}$$

where $\boldsymbol{1}$ is a J-row unit vector. To reiterate, $\boldsymbol{x}_{r,j}$ contains latent group means ($r \leq P$) or manifest level-2 variables ($r > P$). We define $1/\sigma^2_{\zeta_r}$ as a gamma random variable and draw the reciprocal of the residual variance (i.e., the precision) from a gamma distribution

$$1/\sigma^2_{\zeta_r} \sim \mathrm{G}\left(\frac{J + df_p}{2}, \frac{\hat{\boldsymbol{\zeta}}'_{r,j}\hat{\boldsymbol{\zeta}}_{r,j} + S_p}{2}\right) \tag{SA23}$$

with hyperparameters df_p and S_p for the prior distribution. The default setting in Blimp (XPRIOR1) specifies $S_p = 1$ and $df_p = 2$, which corresponds to a gamma(1,.5) prior. Two other options are to set $S_p = 0$ and $df_p = -2$ (the XPRIOR2 keyword of the OPTIONS command) and $S_p = 0$ and $df_p = 0$ (the XPRIOR3 keyword), a

Jeffreys prior. Note keywords induce the same priors at all levels of the data hierarchy.

Step 12: Draw missing values from $p(X_r) \propto p(Y|X_r, X_{-r}) \times p(X_r|X_{-r})$. For each missing score, the Metropolis algorithm draws a candidate imputation $X_{r(candidate)}$ from a normal proposal distribution

$$X^{(t)}_{r(candidate)} = N\left(X^{(t)}_{r(current)}, \sigma^2_{(proposal)}\right) \quad \text{(SA24)}$$

where the mean $X^{(t)}_{r,i(current)}$ is the current imputation i at iteration t, and the variance $\sigma^2_{(proposal)}$ is chosen to optimize the acceptance rate of the candidate imputations. We have found that setting $\sigma^2_{(proposal)} = 9\left(\sigma^2_{e_r}\right)$ for level-1 variables and $\sigma^2_{(proposal)} = 2.25\left(\sigma^2_{\zeta_r}\right)$ for level-2 variables tends to give optimal acceptance rates, although the Blimp application adaptively tunes the spread of the proposal distribution by increasing or decreasing the constant multiplier at regular intervals during the burn-in phase in an attempt to achieve an acceptance rate for the imputations between 0.25 and 0.45 (Gelman et al., 2014; Johnson & Albert, 1999; Lynch, 2007). For each incomplete variable, Blimp checks the acceptance rate every 50 iterations by computing the proportion of accepted imputations for a particular variable across all incomplete observations during the 50-iteration interval (e.g., for 10 incomplete cases, the acceptance rate for a particular variable is the proportion of the 500 draws that are accepted). If the acceptance rate does not fall between 0.25 and 0.45, the program increases (if the acceptance rate is too high) or decreases (if the acceptance rate is too low) the variance multiplier. Once the burn-in iterations are complete, tuning checks are turned off. Normal proposal distributions are routinely used in Bayesian analysis texts (Gelman et al., 2014; Lynch, 2007), but a strong rationale for adopting this distribution in our context is that the correct conditional distribution is, in fact, normal (see manuscript Equations 8 and 20). Thus, the Metropolis algorithm provides a way to model the true distribution of missing values without deriving the complex non-linear functions that define its mean and variance.

After drawing a candidate imputation from the proposal distribution, the Metropolis algorithm calculates the natural logarithm of an importance ratio (IR) that quantifies the height of the target density evaluated at the candidate imputation proportional to its height when evaluated at the current imputation.

$$
\begin{aligned}
ln(\text{IR}) = & \Big[ln[p(Y \mid X_1, \dots, X_{r(candiate)}, \dots, X_R)] + ln[p(X_{r(candiate)}|X_{-r})]\Big] \\
& - \Big[ln[p(Y \mid X_1, \dots, X_{r(current)}, \dots, X_R)] + ln[p(X_{r(current)}|X_{-r})]\Big]
\end{aligned}
\tag{SA25}
$$

Note that $p(Y \mid X_1, \dots, X_{r(candiate)}, \dots, X_R)$ and $(Y \mid X_1, \dots, X_{r(current)}, \dots, X_R)$ involve the product of likelihoods (or the sum of log likelihoods) when X_r is at level-2; $p(X_{r(candiate)}|X_{-r})$ and $p(X_{r(current)}|X_{-r})$ always evaluate a single observation. The importance ratio defines the probability of a Bernoulli random variable that determines whether the candidate value is retained as the current imputation for the next iteration. If the importance ratio exceeds unity, the candidate imputation is automatically accepted. If the ratio is large but less than one, the candidate imputation is likely to be accepted because it has a high probability of originating from $p(Y|X_R) \times p(X_r|X_{-r})$. As the ratio decreases, so too does the chance of retaining the candidate value because it is unlikely to originate from the target density. To account for the natural logarithm, the Metropolis sampler draws a random number u_i from a uniform distribution $U(0,1)$ and accepts $X_{r(candidate)}^{(t)}$ as the new current imputation for the next iteration $t + 1$ if $ln(\text{IR}) > ln(u_i)$.

For incomplete categorical variables, the Metropolis sampler computes the importance ratio as follows

$$\text{IR} = \frac{p(Y|X_1, \dots, X_{r(candiate)}, \dots, X_R)p\left(X^*_{r(candiate)}|X_{-r}\right)}{p(Y|X_1, \dots, X_{r(current)}, \dots, X_R)p\left(X^*_{r(current)}|X_{-r}\right)}. \quad \text{(SA26)}$$

where $X^*_{r(candiate)}$ is a candidate imputation on the underlying latent variable metric. Consistent with the procedure for continuous variables, the algorithm draws a candidate latent variable score from a normal proposal distribution, and the current and candidate synthetic scores, $X^*_{r(current)}$ and $X^*_{r(candiate)}$, respectively, are then evaluated in the $p(X^*_r|X_{-r})$ components of the ratio. The corresponding discrete candidate $X_{r(candiate)}$ for the first component of the numerator product is generated by comparing the latent candidate to the threshold parameter, such that $X_{r(candiate)}$ $= 0$ if $X^*_{r(candiate)} < \kappa$ and $X_{r(candiate)} = 1$ if $X^*_{r(candiate)} \geq \kappa$.

B.

Model-Based Imputation for a Three-Level Model

The three-level analysis model for the third computer simulation is as follows.

$$\begin{aligned} y_{ijk} &= \beta_0 + \beta_1(x_{1ijk}) + \beta_2(x_{2jk}) + \beta_3(x_{3k}) + \beta_4(x_{1ijk})(x_{3k}) \\ &\quad + b_{0k} + b_{1k}(x_{1ijk}) + \; b_{0jk} + b_{1jk}(x_{1ijk}) + \varepsilon_{ijk} \\ \begin{pmatrix} b_{0k} \\ b_{1k} \end{pmatrix} &\sim MN(0, \mathbf{\Sigma}_{b_k}) \begin{pmatrix} b_{0jk} \\ b_{1jk} \end{pmatrix} \sim MN(0, \mathbf{\Sigma}_{b_{jk}}) \;\; \varepsilon_{ijk} \sim N(0, \sigma^2_\varepsilon) \end{aligned} \tag{SB1}$$

Auxiliary variables enter the model as additional explanatory variables.

The covariate distribution is multivariate normal

$$\boldsymbol{x}^{(1)} \sim MN(\boldsymbol{\mu}_{jk}, \mathbf{\Sigma}_1) \;\; \boldsymbol{x}^{(2)} \sim MN(\boldsymbol{\mu}_k, \mathbf{\Sigma}_2) \quad \boldsymbol{x}^{(3)} \sim MN(\boldsymbol{\mu}, \mathbf{\Sigma}_3) \tag{SB2}$$

where $\boldsymbol{x}^{(1)} = (x_{1ijk})$, $\boldsymbol{\mu}_{jk} = (\mu_{1jk})$, $\boldsymbol{x}^{(2)} = (\mu_{1jk}, x_{2jk})$, $\boldsymbol{\mu}_k = (\mu_{1k}, \mu_{2k})$, $\boldsymbol{x}^{(3)} = (\mu_{1k}, \mu_{2k}, x_{3k})$ $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3)$, $\mathbf{\Sigma}_1$ is a scalar within-cluster variance, $\mathbf{\Sigma}_2$ is a 2 by 2 between-cluster covariance matrix at level-2, and $\mathbf{\Sigma}_3$ is a 3 by 3 between-cluster covariance matrix at level-3.

The covariate regression models are as follows, and analogous regressions are constructed for the latent variable cluster means.

$$\begin{aligned} x_{1ijk} &= \mu_{1jk} + e_{1ijk} \\ e_{1ijk} &\sim N(0, \sigma^2_{e_1}) \\ x_{2jk} &= \mu_{2k} + (\mu_{1jk} - \mu_{1k})\eta_{21} + \zeta_{2jk} \\ \zeta_{2jk} &\sim N(0, \sigma^2_{\zeta_2}) \\ x_{3k} &= \mu_3 + (\mu_{1k} - \mu_1)\alpha_{31} + (\mu_{2k} - \mu_2)\alpha_{32} + r_{3k} \\ r_{3k} &\sim N(0, \sigma^2_{r_3}) \end{aligned} \tag{SB3}$$

To make the following expressions more succinct, define $\hat{y}_{ijk}$ as the linear predictor (i.e., predictor value) of Y for observation i in level-2 and level-3 clusters j and k, respectively.

$$\begin{aligned}\hat{y}_{ijk} = \beta_0 + \beta_1(x_{1ijk}) + \beta_2(x_{2jk}) + \beta_3(x_{3k}) + \beta_4(x_{1ijk})(x_{3k}) \\ + b_{0k} + b_{1k}(x_{1ijk}) + \ b_{0jk} + b_{1jk}(x_{1ijk})\end{aligned} \tag{SB4}$$

The Metropolis-Hasting algorithm samples imputations from the following functions

$$\begin{aligned}p(X_1|Y, X_2, X_3) &\propto p(Y\ |X_1, X_2, X_3) \times p(X_1|X_2, X_3) \\ &\propto N(\hat{y}_{ijk}, \sigma^2_\varepsilon) \times N(\mu_{1jk}, \sigma^2_{e_1})\end{aligned} \tag{SB5}$$

$$\begin{aligned}p(X_2|Y, X_1, X_3) &\propto p(Y\ |X_1, X_2, X_3) \times p(X_2|X_1, X_3) \\ &\propto \prod_{i=1}^{n_{i|j}} N(\hat{y}_{ijk}, \sigma^2_\varepsilon) \times N\big(\mu_{2k} + (\mu_{1jk} - \mu_{1k})\eta_{21}, \sigma^2_{\zeta_2}\big)\end{aligned} \tag{SB6}$$

$$\begin{aligned}p(X_3|Y, X_1, X_2) &\propto p(Y\ |X_1, X_2, X_3) \times p(X_3|X_1, X_2) \\ &\propto \prod_{i=1}^{n_{ij|k}} N(\hat{y}_{ijk}, \sigma^2_\varepsilon) \\ &\times N\big(\mu_3 + (\mu_{1k} - \mu_1)\alpha_{31} + (\mu_{2k} - \mu_2)\alpha_{32}, \sigma^2_{r_3}\big)\end{aligned} \tag{SB7}$$

where $n_{i|j}$ denotes all level-1 observations in a given cluster j (i.e., all observations with a particular X_2 score in common), and $n_{ij|k}$ represents the number of observations in a given level-3 cluster k (i.e., all observations with a particular X_3 score in common).

C. Full Results from Computer Simulation Studies

C. Full Results from Computer Simulation Studies

Large Sample Simulation 1: Average Estimates and Bias Values for Model-Based Imputation

ICC	Missing %	Distribution	Parameter	True Value	Avg. Est.	Bias
0.1	15%	Normal	Intercept	50.000	50.002	0.003
0.1	15%	Normal	Level-1 Slope	3.162	3.163	0.020
0.1	15%	Normal	Level-2 Slope	0.744	0.746	0.271
0.1	15%	Normal	Intercept Var.	7.000	6.978	-0.308
0.1	15%	Normal	Icept-Slope Cov.	2.510	2.509	-0.055
0.1	15%	Normal	Slope Var.	10.000	9.988	-0.116
0.1	15%	Normal	Residual Var.	72.000	72.019	0.027
0.1	15%	Skewed	Intercept	50.000	50.006	0.011
0.1	15%	Skewed	Level-1 Slope	3.162	3.158	-0.120
0.1	15%	Skewed	Level-2 Slope	0.744	0.716	-3.767
0.1	15%	Skewed	Intercept Var.	7.000	7.068	0.977
0.1	15%	Skewed	Icept-Slope Cov.	2.510	2.508	-0.090
0.1	15%	Skewed	Slope Var.	10.000	9.983	-0.168
0.1	15%	Skewed	Residual Var.	72.000	71.984	-0.023
0.5	15%	Normal	Intercept	50.000	49.994	-0.012
0.5	15%	Normal	Level-1 Slope	3.162	3.169	0.205
0.5	15%	Normal	Level-2 Slope	1.664	1.673	0.533
0.5	15%	Normal	Intercept Var.	35.000	35.004	0.012
0.5	15%	Normal	Icept-Slope Cov.	5.612	5.546	-1.182
0.5	15%	Normal	Slope Var.	10.000	10.027	0.271
0.5	15%	Normal	Residual Var.	40.000	40.015	0.039
0.5	15%	Skewed	Intercept	50.000	50.020	0.040
0.5	15%	Skewed	Level-1 Slope	3.162	3.149	-0.406
0.5	15%	Skewed	Level-2 Slope	1.664	1.633	-1.822
0.5	15%	Skewed	Intercept Var.	35.000	35.405	1.158
0.5	15%	Skewed	Icept-Slope Cov.	5.612	5.621	0.158
0.5	15%	Skewed	Slope Var.	10.000	10.003	0.034
0.5	15%	Skewed	Residual Var.	40.000	39.989	-0.027
0.1	25%	Normal	Intercept	50.000	49.999	-0.001
0.1	25%	Normal	Level-1 Slope	3.162	3.157	-0.175
0.1	25%	Normal	Level-2 Slope	0.744	0.731	-1.797
0.1	25%	Normal	Intercept Var.	7.000	6.975	-0.353
0.1	25%	Normal	Icept-Slope Cov.	2.510	2.494	-0.653
0.1	25%	Normal	Slope Var.	10.000	9.993	-0.070
0.1	25%	Normal	Residual Var.	72.000	71.996	-0.005
0.1	25%	Skewed	Intercept	50.000	50.011	0.022
0.1	25%	Skewed	Level-1 Slope	3.162	3.161	-0.039
0.1	25%	Skewed	Level-2 Slope	0.744	0.726	-2.470
0.1	25%	Skewed	Intercept Var.	7.000	7.076	1.084
0.1	25%	Skewed	Icept-Slope Cov.	2.510	2.509	-0.038
0.1	25%	Skewed	Slope Var.	10.000	9.995	-0.049

0.1	25%	Skewed	Residual Var.	72.000	71.987	-0.018
0.5	25%	Normal	Intercept	50.000	50.000	0.001
0.5	25%	Normal	Level-1 Slope	3.162	3.151	-0.359
0.5	25%	Normal	Level-2 Slope	1.664	1.646	-1.085
0.5	25%	Normal	Intercept Var.	35.000	35.040	0.114
0.5	25%	Normal	Icept-Slope Cov.	5.612	5.529	-1.480
0.5	25%	Normal	Slope Var.	10.000	10.016	0.164
0.5	25%	Normal	Residual Var.	40.000	40.009	0.022
0.5	25%	Skewed	Intercept	50.000	50.024	0.049
0.5	25%	Skewed	Level-1 Slope	3.162	3.157	-0.183
0.5	25%	Skewed	Level-2 Slope	1.664	1.625	-2.343
0.5	25%	Skewed	Intercept Var.	35.000	35.375	1.071
0.5	25%	Skewed	Icept-Slope Cov.	5.612	5.564	-0.866
0.5	25%	Skewed	Slope Var.	10.000	10.006	0.063
0.5	25%	Skewed	Residual Var.	40.000	39.993	-0.016

Simulation 1: Average Estimates and Bias Values for Model-Based Imputation

N	ICC	Missing %	Distribution	Parameter	True Value	Avg. Est.	Bias
J = 30, n = 10	0.1	15%	Normal	Intercept	50.000	49.987	-0.025
J = 30, n = 30	0.1	15%	Normal	Intercept	50.000	49.971	-0.059
J = 100, n = 10	0.1	15%	Normal	Intercept	50.000	49.991	-0.018
J = 100, n = 30	0.1	15%	Normal	Intercept	50.000	49.993	-0.014
J = 30, n = 10	0.1	15%	Normal	Level-1 Slope	3.162	3.064	-3.096
J = 30, n = 30	0.1	15%	Normal	Level-1 Slope	3.162	3.142	-0.657
J = 100, n = 10	0.1	15%	Normal	Level-1 Slope	3.162	3.119	-1.358
J = 100, n = 30	0.1	15%	Normal	Level-1 Slope	3.162	3.144	-0.569
J = 30, n = 10	0.1	15%	Normal	Level-2 Slope	0.744	0.680	-8.639
J = 30, n = 30	0.1	15%	Normal	Level-2 Slope	0.744	0.674	-9.382
J = 100, n = 10	0.1	15%	Normal	Level-2 Slope	0.744	0.732	-1.565
J = 100, n = 30	0.1	15%	Normal	Level-2 Slope	0.744	0.727	-2.317
J = 30, n = 10	0.1	15%	Normal	Intercept Var.	7.000	6.203	-11.388
J = 30, n = 30	0.1	15%	Normal	Intercept Var.	7.000	6.459	-7.730
J = 100, n = 10	0.1	15%	Normal	Intercept Var.	7.000	6.792	-2.971
J = 100, n = 30	0.1	15%	Normal	Intercept Var.	7.000	6.798	-2.883
J = 30, n = 10	0.1	15%	Normal	Icept-Slope Cov.	2.510	2.162	-13.859
J = 30, n = 30	0.1	15%	Normal	Icept-Slope Cov.	2.510	2.512	0.081
J = 100, n = 10	0.1	15%	Normal	Icept-Slope Cov.	2.510	2.358	-6.057
J = 100, n = 30	0.1	15%	Normal	Icept-Slope Cov.	2.510	2.474	-1.414
J = 30, n = 10	0.1	15%	Normal	Slope Var.	10.000	9.931	-0.686
J = 30, n = 30	0.1	15%	Normal	Slope Var.	10.000	9.761	-2.389
J = 100, n = 10	0.1	15%	Normal	Slope Var.	10.000	9.853	-1.465
J = 100, n = 30	0.1	15%	Normal	Slope Var.	10.000	10.008	0.075
J = 30, n = 10	0.1	15%	Normal	Residual Var.	72.000	71.334	-0.925
J = 30, n = 30	0.1	15%	Normal	Residual Var.	72.000	71.853	-0.204
J = 100, n = 10	0.1	15%	Normal	Residual Var.	72.000	72.124	0.172
J = 100, n = 30	0.1	15%	Normal	Residual Var.	72.000	71.880	-0.167
J = 30, n = 10	0.1	25%	Normal	Intercept	50.000	49.918	-0.164
J = 30, n = 30	0.1	25%	Normal	Intercept	50.000	49.960	-0.079
J = 100, n = 10	0.1	25%	Normal	Intercept	50.000	49.980	-0.041
J = 100, n = 30	0.1	25%	Normal	Intercept	50.000	49.963	-0.073
J = 30, n = 10	0.1	25%	Normal	Level-1 Slope	3.162	2.969	-6.098
J = 30, n = 30	0.1	25%	Normal	Level-1 Slope	3.162	3.116	-1.450
J = 100, n = 10	0.1	25%	Normal	Level-1 Slope	3.162	3.120	-1.322
J = 100, n = 30	0.1	25%	Normal	Level-1 Slope	3.162	3.170	0.245
J = 30, n = 10	0.1	25%	Normal	Level-2 Slope	0.744	0.604	-18.871
J = 30, n = 30	0.1	25%	Normal	Level-2 Slope	0.744	0.668	-10.216
J = 100, n = 10	0.1	25%	Normal	Level-2 Slope	0.744	0.727	-2.286
J = 100, n = 30	0.1	25%	Normal	Level-2 Slope	0.744	0.711	-4.410
J = 30, n = 10	0.1	25%	Normal	Intercept Var.	7.000	6.355	-9.218

J = 30, n = 30	0.1	25%	Normal	Intercept Var.	7.000	6.278	-10.311
J = 100, n = 10	0.1	25%	Normal	Intercept Var.	7.000	6.772	-3.255
J = 100, n = 30	0.1	25%	Normal	Intercept Var.	7.000	6.723	-3.959
J = 30, n = 10	0.1	25%	Normal	Icept-Slope Cov.	2.510	1.731	-31.027
J = 30, n = 30	0.1	25%	Normal	Icept-Slope Cov.	2.510	2.391	-4.745
J = 100, n = 10	0.1	25%	Normal	Icept-Slope Cov.	2.510	2.328	-7.255
J = 100, n = 30	0.1	25%	Normal	Icept-Slope Cov.	2.510	2.466	-1.772
J = 30, n = 10	0.1	25%	Normal	Slope Var.	10.000	10.197	1.975
J = 30, n = 30	0.1	25%	Normal	Slope Var.	10.000	9.782	-2.177
J = 100, n = 10	0.1	25%	Normal	Slope Var.	10.000	10.187	1.869
J = 100, n = 30	0.1	25%	Normal	Slope Var.	10.000	10.042	0.423
J = 30, n = 10	0.1	25%	Normal	Residual Var.	72.000	71.709	-0.404
J = 30, n = 30	0.1	25%	Normal	Residual Var.	72.000	71.959	-0.057
J = 100, n = 10	0.1	25%	Normal	Residual Var.	72.000	71.767	-0.323
J = 100, n = 30	0.1	25%	Normal	Residual Var.	72.000	72.022	0.030
J = 30, n = 10	0.5	15%	Normal	Intercept	50.000	49.908	-0.184
J = 30, n = 30	0.5	15%	Normal	Intercept	50.000	49.967	-0.066
J = 100, n = 10	0.5	15%	Normal	Intercept	50.000	49.975	-0.050
J = 100, n = 30	0.5	15%	Normal	Intercept	50.000	49.946	-0.107
J = 30, n = 10	0.5	15%	Normal	Level-1 Slope	3.162	3.068	-2.987
J = 30, n = 30	0.5	15%	Normal	Level-1 Slope	3.162	3.147	-0.472
J = 100, n = 10	0.5	15%	Normal	Level-1 Slope	3.162	3.137	-0.803
J = 100, n = 30	0.5	15%	Normal	Level-1 Slope	3.162	3.157	-0.164
J = 30, n = 10	0.5	15%	Normal	Level-2 Slope	1.664	1.547	-7.041
J = 30, n = 30	0.5	15%	Normal	Level-2 Slope	1.664	1.566	-5.881
J = 100, n = 10	0.5	15%	Normal	Level-2 Slope	1.664	1.617	-2.779
J = 100, n = 30	0.5	15%	Normal	Level-2 Slope	1.664	1.590	-4.403
J = 30, n = 10	0.5	15%	Normal	Intercept Var.	35.000	32.774	-6.359
J = 30, n = 30	0.5	15%	Normal	Intercept Var.	35.000	32.426	-7.354
J = 100, n = 10	0.5	15%	Normal	Intercept Var.	35.000	34.396	-1.726
J = 100, n = 30	0.5	15%	Normal	Intercept Var.	35.000	34.264	-2.102
J = 30, n = 10	0.5	15%	Normal	Icept-Slope Cov.	5.612	4.946	-11.883
J = 30, n = 30	0.5	15%	Normal	Icept-Slope Cov.	5.612	5.183	-7.657
J = 100, n = 10	0.5	15%	Normal	Icept-Slope Cov.	5.612	5.383	-4.090
J = 100, n = 30	0.5	15%	Normal	Icept-Slope Cov.	5.612	5.487	-2.242
J = 30, n = 10	0.5	15%	Normal	Slope Var.	10.000	9.540	-4.597
J = 30, n = 30	0.5	15%	Normal	Slope Var.	10.000	9.825	-1.755
J = 100, n = 10	0.5	15%	Normal	Slope Var.	10.000	9.905	-0.950
J = 100, n = 30	0.5	15%	Normal	Slope Var.	10.000	10.036	0.356
J = 30, n = 10	0.5	15%	Normal	Residual Var.	40.000	40.105	0.262
J = 30, n = 30	0.5	15%	Normal	Residual Var.	40.000	39.977	-0.058
J = 100, n = 10	0.5	15%	Normal	Residual Var.	40.000	39.975	-0.061
J = 100, n = 30	0.5	15%	Normal	Residual Var.	40.000	39.962	-0.094

J = 30, n = 10	0.5	25%	Normal	Intercept	50.000	49.931	-0.138
J = 30, n = 30	0.5	25%	Normal	Intercept	50.000	49.900	-0.200
J = 100, n = 10	0.5	25%	Normal	Intercept	50.000	49.974	-0.052
J = 100, n = 30	0.5	25%	Normal	Intercept	50.000	49.971	-0.057
J = 30, n = 10	0.5	25%	Normal	Level-1 Slope	3.162	2.995	-5.278
J = 30, n = 30	0.5	25%	Normal	Level-1 Slope	3.162	3.147	-0.468
J = 100, n = 10	0.5	25%	Normal	Level-1 Slope	3.162	3.086	-2.413
J = 100, n = 30	0.5	25%	Normal	Level-1 Slope	3.162	3.148	-0.457
J = 30, n = 10	0.5	25%	Normal	Level-2 Slope	1.664	1.488	-10.585
J = 30, n = 30	0.5	25%	Normal	Level-2 Slope	1.664	1.365	-17.924
J = 100, n = 10	0.5	25%	Normal	Level-2 Slope	1.664	1.561	-6.181
J = 100, n = 30	0.5	25%	Normal	Level-2 Slope	1.664	1.559	-6.322
J = 30, n = 10	0.5	25%	Normal	Intercept Var.	35.000	32.569	-6.945
J = 30, n = 30	0.5	25%	Normal	Intercept Var.	35.000	32.632	-6.765
J = 100, n = 10	0.5	25%	Normal	Intercept Var.	35.000	34.156	-2.413
J = 100, n = 30	0.5	25%	Normal	Intercept Var.	35.000	34.105	-2.557
J = 30, n = 10	0.5	25%	Normal	Icept-Slope Cov.	5.612	4.898	-12.724
J = 30, n = 30	0.5	25%	Normal	Icept-Slope Cov.	5.612	4.926	-12.228
J = 100, n = 10	0.5	25%	Normal	Icept-Slope Cov.	5.612	5.244	-6.572
J = 100, n = 30	0.5	25%	Normal	Icept-Slope Cov.	5.612	5.452	-2.851
J = 30, n = 10	0.5	25%	Normal	Slope Var.	10.000	10.187	1.871
J = 30, n = 30	0.5	25%	Normal	Slope Var.	10.000	9.699	-3.014
J = 100, n = 10	0.5	25%	Normal	Slope Var.	10.000	10.202	2.022
J = 100, n = 30	0.5	25%	Normal	Slope Var.	10.000	10.050	0.501
J = 30, n = 10	0.5	25%	Normal	Residual Var.	40.000	39.954	-0.116
J = 30, n = 30	0.5	25%	Normal	Residual Var.	40.000	39.990	-0.025
J = 100, n = 10	0.5	25%	Normal	Residual Var.	40.000	39.986	-0.035
J = 100, n = 30	0.5	25%	Normal	Residual Var.	40.000	40.005	0.013
J = 30, n = 10	0.1	15%	Skewed	Intercept	50.000	49.954	-0.092
J = 30, n = 30	0.1	15%	Skewed	Intercept	50.000	49.972	-0.056
J = 100, n = 10	0.1	15%	Skewed	Intercept	50.000	49.988	-0.024
J = 100, n = 30	0.1	15%	Skewed	Intercept	50.000	50.002	0.005
J = 30, n = 10	0.1	15%	Skewed	Level-1 Slope	3.162	3.095	-2.142
J = 30, n = 30	0.1	15%	Skewed	Level-1 Slope	3.162	3.145	-0.547
J = 100, n = 10	0.1	15%	Skewed	Level-1 Slope	3.162	3.150	-0.396
J = 100, n = 30	0.1	15%	Skewed	Level-1 Slope	3.162	3.165	0.090
J = 30, n = 10	0.1	15%	Skewed	Level-2 Slope	0.744	0.714	-3.995
J = 30, n = 30	0.1	15%	Skewed	Level-2 Slope	0.744	0.681	-8.427
J = 100, n = 10	0.1	15%	Skewed	Level-2 Slope	0.744	0.719	-3.371
J = 100, n = 30	0.1	15%	Skewed	Level-2 Slope	0.744	0.722	-3.001
J = 30, n = 10	0.1	15%	Skewed	Intercept Var.	7.000	6.481	-7.420
J = 30, n = 30	0.1	15%	Skewed	Intercept Var.	7.000	6.506	-7.051
J = 100, n = 10	0.1	15%	Skewed	Intercept Var.	7.000	6.695	-4.351

J = 100, n = 30	0.1	15% Skewed	Intercept Var.	7.000	6.860	-2.005
J = 30, n = 10	0.1	15% Skewed	Icept-Slope Cov.	2.510	2.220	-11.551
J = 30, n = 30	0.1	15% Skewed	Icept-Slope Cov.	2.510	2.400	-4.376
J = 100, n = 10	0.1	15% Skewed	Icept-Slope Cov.	2.510	2.512	0.094
J = 100, n = 30	0.1	15% Skewed	Icept-Slope Cov.	2.510	2.449	-2.443
J = 30, n = 10	0.1	15% Skewed	Slope Var.	10.000	9.936	-0.645
J = 30, n = 30	0.1	15% Skewed	Slope Var.	10.000	9.727	-2.733
J = 100, n = 10	0.1	15% Skewed	Slope Var.	10.000	10.136	1.357
J = 100, n = 30	0.1	15% Skewed	Slope Var.	10.000	9.826	-1.744
J = 30, n = 10	0.1	15% Skewed	Residual Var.	72.000	71.119	-1.224
J = 30, n = 30	0.1	15% Skewed	Residual Var.	72.000	72.040	0.055
J = 100, n = 10	0.1	15% Skewed	Residual Var.	72.000	71.994	-0.009
J = 100, n = 30	0.1	15% Skewed	Residual Var.	72.000	72.156	0.217
J = 30, n = 10	0.1	25% Skewed	Intercept	50.000	50.003	0.007
J = 30, n = 30	0.1	25% Skewed	Intercept	50.000	49.950	-0.100
J = 100, n = 10	0.1	25% Skewed	Intercept	50.000	49.954	-0.091
J = 100, n = 30	0.1	25% Skewed	Intercept	50.000	50.008	0.015
J = 30, n = 10	0.1	25% Skewed	Level-1 Slope	3.162	3.024	-4.358
J = 30, n = 30	0.1	25% Skewed	Level-1 Slope	3.162	3.157	-0.157
J = 100, n = 10	0.1	25% Skewed	Level-1 Slope	3.162	3.132	-0.949
J = 100, n = 30	0.1	25% Skewed	Level-1 Slope	3.162	3.169	0.207
J = 30, n = 10	0.1	25% Skewed	Level-2 Slope	0.744	0.606	-18.528
J = 30, n = 30	0.1	25% Skewed	Level-2 Slope	0.744	0.615	-17.334
J = 100, n = 10	0.1	25% Skewed	Level-2 Slope	0.744	0.663	-10.841
J = 100, n = 30	0.1	25% Skewed	Level-2 Slope	0.744	0.689	-7.381
J = 30, n = 10	0.1	25% Skewed	Intercept Var.	7.000	6.418	-8.309
J = 30, n = 30	0.1	25% Skewed	Intercept Var.	7.000	6.443	-7.963
J = 100, n = 10	0.1	25% Skewed	Intercept Var.	7.000	6.690	-4.434
J = 100, n = 30	0.1	25% Skewed	Intercept Var.	7.000	6.783	-3.107
J = 30, n = 10	0.1	25% Skewed	Icept-Slope Cov.	2.510	2.076	-17.299
J = 30, n = 30	0.1	25% Skewed	Icept-Slope Cov.	2.510	2.402	-4.321
J = 100, n = 10	0.1	25% Skewed	Icept-Slope Cov.	2.510	2.380	-5.166
J = 100, n = 30	0.1	25% Skewed	Icept-Slope Cov.	2.510	2.496	-0.549
J = 30, n = 10	0.1	25% Skewed	Slope Var.	10.000	10.390	3.900
J = 30, n = 30	0.1	25% Skewed	Slope Var.	10.000	9.723	-2.767
J = 100, n = 10	0.1	25% Skewed	Slope Var.	10.000	10.016	0.161
J = 100, n = 30	0.1	25% Skewed	Slope Var.	10.000	9.853	-1.465
J = 30, n = 10	0.1	25% Skewed	Residual Var.	72.000	71.625	-0.521
J = 30, n = 30	0.1	25% Skewed	Residual Var.	72.000	71.989	-0.016
J = 100, n = 10	0.1	25% Skewed	Residual Var.	72.000	72.067	0.094
J = 100, n = 30	0.1	25% Skewed	Residual Var.	72.000	71.985	-0.020
J = 30, n = 10	0.5	15% Skewed	Intercept	50.000	49.946	-0.108
J = 30, n = 30	0.5	15% Skewed	Intercept	50.000	49.972	-0.057

J = 100, n = 10	0.5	15% Skewed	Intercept	50.000	49.976	-0.048
J = 100, n = 30	0.5	15% Skewed	Intercept	50.000	50.028	0.057
J = 30, n = 10	0.5	15% Skewed	Level-1 Slope	3.162	3.092	-2.217
J = 30, n = 30	0.5	15% Skewed	Level-1 Slope	3.162	3.136	-0.818
J = 100, n = 10	0.5	15% Skewed	Level-1 Slope	3.162	3.123	-1.229
J = 100, n = 30	0.5	15% Skewed	Level-1 Slope	3.162	3.161	-0.055
J = 30, n = 10	0.5	15% Skewed	Level-2 Slope	1.664	1.547	-7.000
J = 30, n = 30	0.5	15% Skewed	Level-2 Slope	1.664	1.539	-7.523
J = 100, n = 10	0.5	15% Skewed	Level-2 Slope	1.664	1.539	-7.473
J = 100, n = 30	0.5	15% Skewed	Level-2 Slope	1.664	1.629	-2.071
J = 30, n = 10	0.5	15% Skewed	Intercept Var.	35.000	32.958	-5.835
J = 30, n = 30	0.5	15% Skewed	Intercept Var.	35.000	33.120	-5.372
J = 100, n = 10	0.5	15% Skewed	Intercept Var.	35.000	34.516	-1.383
J = 100, n = 30	0.5	15% Skewed	Intercept Var.	35.000	34.600	-1.142
J = 30, n = 10	0.5	15% Skewed	Icept-Slope Cov.	5.612	4.952	-11.771
J = 30, n = 30	0.5	15% Skewed	Icept-Slope Cov.	5.612	5.426	-3.317
J = 100, n = 10	0.5	15% Skewed	Icept-Slope Cov.	5.612	5.356	-4.572
J = 100, n = 30	0.5	15% Skewed	Icept-Slope Cov.	5.612	5.508	-1.863
J = 30, n = 10	0.5	15% Skewed	Slope Var.	10.000	9.946	-0.545
J = 30, n = 30	0.5	15% Skewed	Slope Var.	10.000	9.868	-1.319
J = 100, n = 10	0.5	15% Skewed	Slope Var.	10.000	9.700	-2.999
J = 100, n = 30	0.5	15% Skewed	Slope Var.	10.000	9.899	-1.013
J = 30, n = 10	0.5	15% Skewed	Residual Var.	40.000	39.917	-0.208
J = 30, n = 30	0.5	15% Skewed	Residual Var.	40.000	39.902	-0.246
J = 100, n = 10	0.5	15% Skewed	Residual Var.	40.000	39.964	-0.090
J = 100, n = 30	0.5	15% Skewed	Residual Var.	40.000	40.029	0.072
J = 30, n = 10	0.5	25% Skewed	Intercept	50.000	49.905	-0.189
J = 30, n = 30	0.5	25% Skewed	Intercept	50.000	49.981	-0.038
J = 100, n = 10	0.5	25% Skewed	Intercept	50.000	50.010	0.020
J = 100, n = 30	0.5	25% Skewed	Intercept	50.000	49.997	-0.005
J = 30, n = 10	0.5	25% Skewed	Level-1 Slope	3.162	3.037	-3.964
J = 30, n = 30	0.5	25% Skewed	Level-1 Slope	3.162	3.139	-0.742
J = 100, n = 10	0.5	25% Skewed	Level-1 Slope	3.162	3.106	-1.764
J = 100, n = 30	0.5	25% Skewed	Level-1 Slope	3.162	3.175	0.390
J = 30, n = 10	0.5	25% Skewed	Level-2 Slope	1.664	1.444	-13.215
J = 30, n = 30	0.5	25% Skewed	Level-2 Slope	1.664	1.470	-11.620
J = 100, n = 10	0.5	25% Skewed	Level-2 Slope	1.664	1.577	-5.215
J = 100, n = 30	0.5	25% Skewed	Level-2 Slope	1.664	1.554	-6.585
J = 30, n = 10	0.5	25% Skewed	Intercept Var.	35.000	33.290	-4.887
J = 30, n = 30	0.5	25% Skewed	Intercept Var.	35.000	32.789	-6.318
J = 100, n = 10	0.5	25% Skewed	Intercept Var.	35.000	34.758	-0.692
J = 100, n = 30	0.5	25% Skewed	Intercept Var.	35.000	34.776	-0.639
J = 30, n = 10	0.5	25% Skewed	Icept-Slope Cov.	5.612	4.649	-17.173

J = 30, n = 30	0.5	25%	Skewed	Icept-Slope Cov.	5.612	5.119	-8.789
J = 100, n = 10	0.5	25%	Skewed	Icept-Slope Cov.	5.612	5.254	-6.384
J = 100, n = 30	0.5	25%	Skewed	Icept-Slope Cov.	5.612	5.413	-3.555
J = 30, n = 10	0.5	25%	Skewed	Slope Var.	10.000	10.455	4.549
J = 30, n = 30	0.5	25%	Skewed	Slope Var.	10.000	10.021	0.212
J = 100, n = 10	0.5	25%	Skewed	Slope Var.	10.000	10.010	0.102
J = 100, n = 30	0.5	25%	Skewed	Slope Var.	10.000	9.833	-1.665
J = 30, n = 10	0.5	25%	Skewed	Residual Var.	40.000	39.811	-0.472
J = 30, n = 30	0.5	25%	Skewed	Residual Var.	40.000	39.910	-0.224
J = 100, n = 10	0.5	25%	Skewed	Residual Var.	40.000	39.915	-0.213
J = 100, n = 30	0.5	25%	Skewed	Residual Var.	40.000	39.967	-0.083

Simulation 2: Average Estimates and Bias Values for Model-Based Imputation

N	ICC	Missing %	Parameter	rue Value	Avg. Est.	Bias
J = 30, n = 10	0.1	15%	Intercept	50.000	50.073	0.147
J = 30, n = 30	0.1	15%	Intercept	50.000	50.044	0.088
J = 100, n = 10	0.1	15%	Intercept	50.000	50.034	0.068
J = 100, n = 30	0.1	15%	Intercept	50.000	50.006	0.012
J = 30, n = 10	0.1	15%	Level-1 Slope	3.162	3.052	-3.481
J = 30, n = 30	0.1	15%	Level-1 Slope	3.162	3.111	-1.611
J = 100, n = 10	0.1	15%	Level-1 Slope	3.162	3.108	-1.710
J = 100, n = 30	0.1	15%	Level-1 Slope	3.162	3.153	-0.296
J = 30, n = 10	0.1	15%	Level-2 Slope	0.861	0.711	-17.403
J = 30, n = 30	0.1	15%	Level-2 Slope	0.861	0.739	-14.163
J = 100, n = 10	0.1	15%	Level-2 Slope	0.861	0.797	-7.487
J = 100, n = 30	0.1	15%	Level-2 Slope	0.861	0.816	-5.265
J = 30, n = 10	0.1	15%	Intercept Var.	7.556	6.839	-9.495
J = 30, n = 30	0.1	15%	Intercept Var.	7.556	6.942	-8.130
J = 100, n = 10	0.1	15%	Intercept Var.	7.556	7.238	-4.207
J = 100, n = 30	0.1	15%	Intercept Var.	7.556	7.421	-1.783
J = 30, n = 10	0.1	15%	Icept-Slope Cov.	2.608	2.071	-20.583
J = 30, n = 30	0.1	15%	Icept-Slope Cov.	2.608	2.476	-5.073
J = 100, n = 10	0.1	15%	Icept-Slope Cov.	2.608	2.498	-4.223
J = 100, n = 30	0.1	15%	Icept-Slope Cov.	2.608	2.599	-0.333
J = 30, n = 10	0.1	15%	Slope Var.	10.000	9.824	-1.761
J = 30, n = 30	0.1	15%	Slope Var.	10.000	9.734	-2.660
J = 100, n = 10	0.1	15%	Slope Var.	10.000	9.921	-0.785
J = 100, n = 30	0.1	15%	Slope Var.	10.000	9.778	-2.222
J = 30, n = 10	0.1	15%	Residual Var.	72.000	71.452	-0.761
J = 30, n = 30	0.1	15%	Residual Var.	72.000	71.900	-0.139
J = 100, n = 10	0.1	15%	Residual Var.	72.000	72.016	0.022
J = 100, n = 30	0.1	15%	Residual Var.	72.000	71.873	-0.176
J = 30, n = 10	0.1	25%	Intercept	50.000	50.045	0.090
J = 30, n = 30	0.1	25%	Intercept	50.000	50.008	0.016
J = 100, n = 10	0.1	25%	Intercept	50.000	50.001	0.002
J = 100, n = 30	0.1	25%	Intercept	50.000	50.015	0.030
J = 30, n = 10	0.1	25%	Level-1 Slope	3.162	3.084	-2.475
J = 30, n = 30	0.1	25%	Level-1 Slope	3.162	3.137	-0.788
J = 100, n = 10	0.1	25%	Level-1 Slope	3.162	3.099	-1.989
J = 100, n = 30	0.1	25%	Level-1 Slope	3.162	3.162	-0.002
J = 30, n = 10	0.1	25%	Level-2 Slope	0.861	0.704	-18.248
J = 30, n = 30	0.1	25%	Level-2 Slope	0.861	0.759	-11.823
J = 100, n = 10	0.1	25%	Level-2 Slope	0.861	0.840	-2.514
J = 100, n = 30	0.1	25%	Level-2 Slope	0.861	0.815	-5.358
J = 30, n = 10	0.1	25%	Intercept Var.	7.556	6.756	-10.593

J = 30, n = 30	0.1	25%	Intercept Var.	7.556	6.822	-9.718
J = 100, n = 10	0.1	25%	Intercept Var.	7.556	7.160	-5.245
J = 100, n = 30	0.1	25%	Intercept Var.	7.556	7.338	-2.886
J = 30, n = 10	0.1	25%	Icept-Slope Cov.	2.608	2.260	-13.340
J = 30, n = 30	0.1	25%	Icept-Slope Cov.	2.608	2.523	-3.256
J = 100, n = 10	0.1	25%	Icept-Slope Cov.	2.608	2.438	-6.501
J = 100, n = 30	0.1	25%	Icept-Slope Cov.	2.608	2.587	-0.809
J = 30, n = 10	0.1	25%	Slope Var.	10.000	10.530	5.300
J = 30, n = 30	0.1	25%	Slope Var.	10.000	9.860	-1.404
J = 100, n = 10	0.1	25%	Slope Var.	10.000	9.960	-0.396
J = 100, n = 30	0.1	25%	Slope Var.	10.000	10.077	0.774
J = 30, n = 10	0.1	25%	Residual Var.	72.000	71.558	-0.613
J = 30, n = 30	0.1	25%	Residual Var.	72.000	71.715	-0.396
J = 100, n = 10	0.1	25%	Residual Var.	72.000	71.942	-0.081
J = 100, n = 30	0.1	25%	Residual Var.	72.000	71.920	-0.112
J = 30, n = 10	0.5	15%	Intercept	50.000	49.956	-0.088
J = 30, n = 30	0.5	15%	Intercept	50.000	50.009	0.017
J = 100, n = 10	0.5	15%	Intercept	50.000	50.043	0.085
J = 100, n = 30	0.5	15%	Intercept	50.000	50.041	0.083
J = 30, n = 10	0.5	15%	Level-1 Slope	3.162	3.071	-2.880
J = 30, n = 30	0.5	15%	Level-1 Slope	3.162	3.174	0.362
J = 100, n = 10	0.5	15%	Level-1 Slope	3.162	3.148	-0.448
J = 100, n = 30	0.5	15%	Level-1 Slope	3.162	3.169	0.224
J = 30, n = 10	0.5	15%	Level-2 Slope	1.926	1.865	-3.162
J = 30, n = 30	0.5	15%	Level-2 Slope	1.926	1.845	-4.207
J = 100, n = 10	0.5	15%	Level-2 Slope	1.926	1.816	-5.686
J = 100, n = 30	0.5	15%	Level-2 Slope	1.926	1.799	-6.596
J = 30, n = 10	0.5	15%	Intercept Var.	37.781	34.949	-7.495
J = 30, n = 30	0.5	15%	Intercept Var.	37.781	35.002	-7.355
J = 100, n = 10	0.5	15%	Intercept Var.	37.781	36.671	-2.939
J = 100, n = 30	0.5	15%	Intercept Var.	37.781	36.730	-2.784
J = 30, n = 10	0.5	15%	Icept-Slope Cov.	5.831	4.883	-16.255
J = 30, n = 30	0.5	15%	Icept-Slope Cov.	5.831	5.555	-4.736
J = 100, n = 10	0.5	15%	Icept-Slope Cov.	5.831	5.619	-3.635
J = 100, n = 30	0.5	15%	Icept-Slope Cov.	5.831	5.713	-2.020
J = 30, n = 10	0.5	15%	Slope Var.	10.000	9.847	-1.535
J = 30, n = 30	0.5	15%	Slope Var.	10.000	9.691	-3.091
J = 100, n = 10	0.5	15%	Slope Var.	10.000	9.998	-0.020
J = 100, n = 30	0.5	15%	Slope Var.	10.000	9.979	-0.210
J = 30, n = 10	0.5	15%	Residual Var.	40.000	40.050	0.125
J = 30, n = 30	0.5	15%	Residual Var.	40.000	39.824	-0.441
J = 100, n = 10	0.5	15%	Residual Var.	40.000	39.987	-0.032
J = 100, n = 30	0.5	15%	Residual Var.	40.000	40.015	0.038

J = 30, n = 10	0.5	25%	Intercept	50.000	50.068	0.135
J = 30, n = 30	0.5	25%	Intercept	50.000	50.045	0.090
J = 100, n = 10	0.5	25%	Intercept	50.000	50.052	0.105
J = 100, n = 30	0.5	25%	Intercept	50.000	49.984	-0.031
J = 30, n = 10	0.5	25%	Level-1 Slope	3.162	3.061	-3.216
J = 30, n = 30	0.5	25%	Level-1 Slope	3.162	3.119	-1.358
J = 100, n = 10	0.5	25%	Level-1 Slope	3.162	3.123	-1.241
J = 100, n = 30	0.5	25%	Level-1 Slope	3.162	3.107	-1.741
J = 30, n = 10	0.5	25%	Level-2 Slope	1.926	1.568	-18.571
J = 30, n = 30	0.5	25%	Level-2 Slope	1.926	1.581	-17.917
J = 100, n = 10	0.5	25%	Level-2 Slope	1.926	1.756	-8.829
J = 100, n = 30	0.5	25%	Level-2 Slope	1.926	1.832	-4.865
J = 30, n = 10	0.5	25%	Intercept Var.	37.781	34.805	-7.877
J = 30, n = 30	0.5	25%	Intercept Var.	37.781	34.800	-7.890
J = 100, n = 10	0.5	25%	Intercept Var.	37.781	36.606	-3.110
J = 100, n = 30	0.5	25%	Intercept Var.	37.781	37.031	-1.984
J = 30, n = 10	0.5	25%	Icept-Slope Cov.	5.831	4.736	-18.778
J = 30, n = 30	0.5	25%	Icept-Slope Cov.	5.831	5.327	-8.647
J = 100, n = 10	0.5	25%	Icept-Slope Cov.	5.831	5.582	-4.269
J = 100, n = 30	0.5	25%	Icept-Slope Cov.	5.831	5.624	-3.561
J = 30, n = 10	0.5	25%	Slope Var.	10.000	10.240	2.400
J = 30, n = 30	0.5	25%	Slope Var.	10.000	9.891	-1.088
J = 100, n = 10	0.5	25%	Slope Var.	10.000	10.004	0.037
J = 100, n = 30	0.5	25%	Slope Var.	10.000	9.936	-0.639
J = 30, n = 10	0.5	25%	Residual Var.	40.000	40.025	0.062
J = 30, n = 30	0.5	25%	Residual Var.	40.000	39.928	-0.181
J = 100, n = 10	0.5	25%	Residual Var.	40.000	40.076	0.190
J = 100, n = 30	0.5	25%	Residual Var.	40.000	40.026	0.064

Simulation 3: Average Estimates and Bias Values for Model-Based Imputation

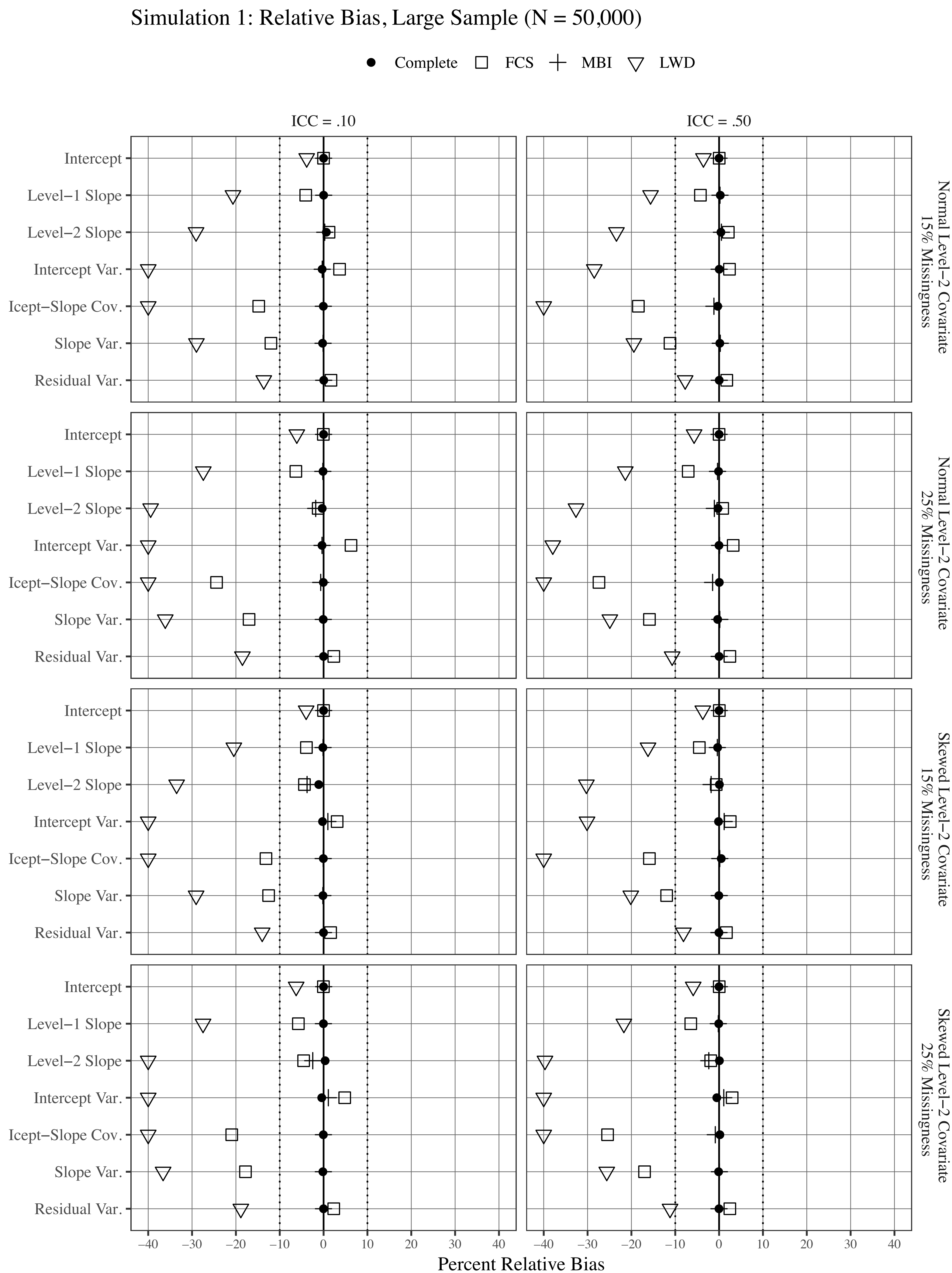
N	ICC	Missing %	Parameter	True Value	Avg. Est.	Bias
J = 30, n = 10	0.1	15%	Intercept	49.836	49.819	-0.033
J = 30, n = 10	0.1	15%	Level-1 Slope	3.098	3.099	0.023
J = 30, n = 10	0.1	15%	Level-2 Slope	0.724	0.706	-2.495
J = 30, n = 10	0.1	15%	Level-3 Slope	0.654	0.630	-3.644
J = 30, n = 10	0.1	15%	Interaction Slope	1.549	1.436	-7.276
J = 30, n = 10	0.1	15%	Level-3 Intercept Var.	5.104	4.557	-10.711
J = 30, n = 10	0.1	15%	Level-3 Icept-Slope Cov.	1.917	1.893	-1.231
J = 30, n = 10	0.1	15%	Level-3 Slope Var.	8.000	7.436	-7.055
J = 30, n = 10	0.1	15%	Level-2 Intercept Var.	5.500	5.487	-0.236
J = 30, n = 10	0.1	15%	Level-2 Icept-Slope Cov.	1.990	2.015	1.271
J = 30, n = 10	0.1	15%	Level-2 Slope Var.	8.000	8.096	1.205
J = 30, n = 10	0.1	15%	Residual Var.	52.000	51.989	-0.020
J = 30, n = 10	0.5	15%	Intercept	49.740	49.723	-0.034
J = 30, n = 10	0.5	15%	Level-1 Slope	2.449	2.437	-0.505
J = 30, n = 10	0.5	15%	Level-2 Slope	1.145	1.132	-1.098
J = 30, n = 10	0.5	15%	Level-3 Slope	0.938	0.857	-8.628
J = 30, n = 10	0.5	15%	Interaction Slope	1.225	1.138	-7.066
J = 30, n = 10	0.5	15%	Level-3 Intercept Var.	11.183	10.208	-8.718
J = 30, n = 10	0.5	15%	Level-3 Icept-Slope Cov.	2.243	1.985	-11.523
J = 30, n = 10	0.5	15%	Level-3 Slope Var.	5.000	4.618	-7.641
J = 30, n = 10	0.5	15%	Level-2 Intercept Var.	13.750	13.649	-0.732
J = 30, n = 10	0.5	15%	Level-2 Icept-Slope Cov.	2.487	2.353	-5.418
J = 30, n = 10	0.5	15%	Level-2 Slope Var.	5.000	5.113	2.261
J = 30, n = 10	0.5	15%	Residual Var.	32.500	32.548	0.148
J = 30, n = 10	0.1	25%	Intercept	49.836	49.818	-0.036
J = 30, n = 10	0.1	25%	Level-1 Slope	3.098	3.089	-0.297
J = 30, n = 10	0.1	25%	Level-2 Slope	0.724	0.700	-3.341
J = 30, n = 10	0.1	25%	Level-3 Slope	0.654	0.513	-21.541
J = 30, n = 10	0.1	25%	Interaction Slope	1.549	1.367	-11.749
J = 30, n = 10	0.1	25%	Level-3 Intercept Var.	5.104	4.647	-8.962
J = 30, n = 10	0.1	25%	Level-3 Icept-Slope Cov.	1.917	1.933	0.828
J = 30, n = 10	0.1	25%	Level-3 Slope Var.	8.000	7.560	-5.503
J = 30, n = 10	0.1	25%	Level-2 Intercept Var.	5.500	5.450	-0.908
J = 30, n = 10	0.1	25%	Level-2 Icept-Slope Cov.	1.990	1.983	-0.358
J = 30, n = 10	0.1	25%	Level-2 Slope Var.	8.000	8.219	2.738
J = 30, n = 10	0.1	25%	Residual Var.	52.000	51.964	-0.070
J = 30, n = 10	0.5	25%	Intercept	49.740	49.694	-0.092
J = 30, n = 10	0.5	25%	Level-1 Slope	2.449	2.403	-1.886
J = 30, n = 10	0.5	25%	Level-2 Slope	1.145	1.116	-2.517
J = 30, n = 10	0.5	25%	Level-3 Slope	0.938	0.745	-20.550
J = 30, n = 10	0.5	25%	Interaction Slope	1.225	1.057	-13.668

J = 30, n = 10	0.5	25%	Level-3 Intercept Var.	11.183	10.546	-5.694
J = 30, n = 10	0.5	25%	Level-3 Icept-Slope Cov.	2.243	2.157	-3.832
J = 30, n = 10	0.5	25%	Level-3 Slope Var.	5.000	4.748	-5.038
J = 30, n = 10	0.5	25%	Level-2 Intercept Var.	13.750	13.852	0.739
J = 30, n = 10	0.5	25%	Level-2 Icept-Slope Cov.	2.487	2.273	-8.625
J = 30, n = 10	0.5	25%	Level-2 Slope Var.	5.000	5.124	2.473
J = 30, n = 10	0.5	25%	Residual Var.	32.500	32.570	0.217
J = 100, n = 10	0.1	15%	Intercept	49.836	49.835	-0.002
J = 100, n = 10	0.1	15%	Level-1 Slope	3.098	3.099	0.017
J = 100, n = 10	0.1	15%	Level-2 Slope	0.724	0.726	0.322
J = 100, n = 10	0.1	15%	Level-3 Slope	0.654	0.620	-5.249
J = 100, n = 10	0.1	15%	Interaction Slope	1.549	1.490	-3.808
J = 100, n = 10	0.1	15%	Level-3 Intercept Var.	5.104	4.914	-3.714
J = 100, n = 10	0.1	15%	Level-3 Icept-Slope Cov.	1.917	1.903	-0.730
J = 100, n = 10	0.1	15%	Level-3 Slope Var.	8.000	7.817	-2.286
J = 100, n = 10	0.1	15%	Level-2 Intercept Var.	5.500	5.453	-0.863
J = 100, n = 10	0.1	15%	Level-2 Icept-Slope Cov.	1.990	1.981	-0.432
J = 100, n = 10	0.1	15%	Level-2 Slope Var.	8.000	7.970	-0.370
J = 100, n = 10	0.1	15%	Residual Var.	52.000	52.039	0.075
J = 100, n = 10	0.5	15%	Intercept	49.740	49.754	0.027
J = 100, n = 10	0.5	15%	Level-1 Slope	2.449	2.463	0.550
J = 100, n = 10	0.5	15%	Level-2 Slope	1.145	1.143	-0.154
J = 100, n = 10	0.5	15%	Level-3 Slope	0.938	0.876	-6.664
J = 100, n = 10	0.5	15%	Interaction Slope	1.225	1.179	-3.723
J = 100, n = 10	0.5	15%	Level-3 Intercept Var.	11.183	10.995	-1.674
J = 100, n = 10	0.5	15%	Level-3 Icept-Slope Cov.	2.243	2.230	-0.605
J = 100, n = 10	0.5	15%	Level-3 Slope Var.	5.000	4.922	-1.552
J = 100, n = 10	0.5	15%	Level-2 Intercept Var.	13.750	13.697	-0.383
J = 100, n = 10	0.5	15%	Level-2 Icept-Slope Cov.	2.487	2.440	-1.919
J = 100, n = 10	0.5	15%	Level-2 Slope Var.	5.000	5.076	1.516
J = 100, n = 10	0.5	15%	Residual Var.	32.500	32.497	-0.008
J = 100, n = 10	0.1	25%	Intercept	49.836	49.836	0.001
J = 100, n = 10	0.1	25%	Level-1 Slope	3.098	3.096	-0.080
J = 100, n = 10	0.1	25%	Level-2 Slope	0.724	0.709	-2.116
J = 100, n = 10	0.1	25%	Level-3 Slope	0.654	0.608	-7.039
J = 100, n = 10	0.1	25%	Interaction Slope	1.549	1.462	-5.605
J = 100, n = 10	0.1	25%	Level-3 Intercept Var.	5.104	4.985	-2.342
J = 100, n = 10	0.1	25%	Level-3 Icept-Slope Cov.	1.917	2.002	4.454
J = 100, n = 10	0.1	25%	Level-3 Slope Var.	8.000	8.017	0.208
J = 100, n = 10	0.1	25%	Level-2 Intercept Var.	5.500	5.500	0.002
J = 100, n = 10	0.1	25%	Level-2 Icept-Slope Cov.	1.990	1.941	-2.459
J = 100, n = 10	0.1	25%	Level-2 Slope Var.	8.000	8.065	0.818
J = 100, n = 10	0.1	25%	Residual Var.	52.000	52.030	0.058

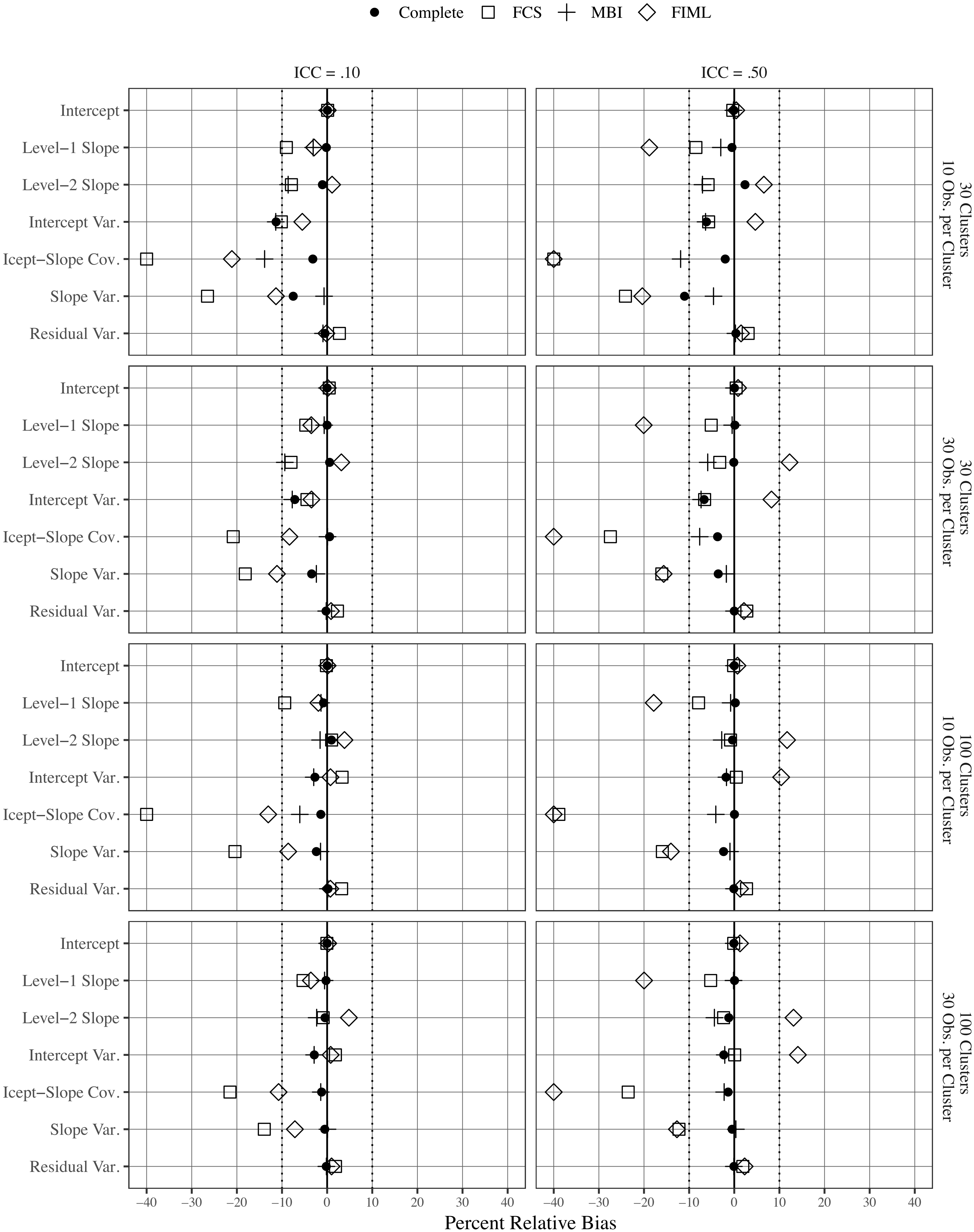
J = 100, n = 10	0.5	25%	Intercept	49.740	49.749	0.018
J = 100, n = 10	0.5	25%	Level-1 Slope	2.449	2.430	-0.777
J = 100, n = 10	0.5	25%	Level-2 Slope	1.145	1.121	-2.058
J = 100, n = 10	0.5	25%	Level-3 Slope	0.938	0.850	-9.458
J = 100, n = 10	0.5	25%	Interaction Slope	1.225	1.136	-7.220
J = 100, n = 10	0.5	25%	Level-3 Intercept Var.	11.183	11.137	-0.408
J = 100, n = 10	0.5	25%	Level-3 Icept-Slope Cov.	2.243	2.198	-2.010
J = 100, n = 10	0.5	25%	Level-3 Slope Var.	5.000	5.038	0.769
J = 100, n = 10	0.5	25%	Level-2 Intercept Var.	13.750	13.850	0.729
J = 100, n = 10	0.5	25%	Level-2 Icept-Slope Cov.	2.487	2.413	-2.999
J = 100, n = 10	0.5	25%	Level-2 Slope Var.	5.000	5.119	2.381
J = 100, n = 10	0.5	25%	Residual Var.	32.500	32.508	0.026
J = 30, n = 30	0.1	15%	Intercept	49.836	49.837	0.002
J = 30, n = 30	0.1	15%	Level-1 Slope	3.098	3.102	0.123
J = 30, n = 30	0.1	15%	Level-2 Slope	0.724	0.713	-1.575
J = 30, n = 30	0.1	15%	Level-3 Slope	0.654	0.591	-9.673
J = 30, n = 30	0.1	15%	Interaction Slope	1.549	1.449	-6.463
J = 30, n = 30	0.1	15%	Level-3 Intercept Var.	5.104	4.629	-9.300
J = 30, n = 30	0.1	15%	Level-3 Icept-Slope Cov.	1.917	1.843	-3.881
J = 30, n = 30	0.1	15%	Level-3 Slope Var.	8.000	7.447	-6.908
J = 30, n = 30	0.1	15%	Level-2 Intercept Var.	5.500	5.498	-0.035
J = 30, n = 30	0.1	15%	Level-2 Icept-Slope Cov.	1.990	2.001	0.550
J = 30, n = 30	0.1	15%	Level-2 Slope Var.	8.000	7.995	-0.057
J = 30, n = 30	0.1	15%	Residual Var.	52.000	52.053	0.101
J = 30, n = 30	0.5	15%	Intercept	49.740	49.798	0.117
J = 30, n = 30	0.5	15%	Level-1 Slope	2.449	2.439	-0.432
J = 30, n = 30	0.5	15%	Level-2 Slope	1.145	1.139	-0.499
J = 30, n = 30	0.5	15%	Level-3 Slope	0.938	0.880	-6.174
J = 30, n = 30	0.5	0.15	Interaction Slope	1.225	1.083	-11.542
J = 30, n = 30	0.5	0.15	Level-3 Intercept Var.	11.183	10.421	-6.811
J = 30, n = 30	0.5	0.15	Level-3 Icept-Slope Cov.	2.243	2.148	-4.266
J = 30, n = 30	0.5	0.15	Level-3 Slope Var.	5.000	4.444	-11.113
J = 30, n = 30	0.5	0.15	Level-2 Intercept Var.	13.750	14.550	5.815
J = 30, n = 30	0.5	0.15	Level-2 Icept-Slope Cov.	2.487	2.053	-17.481
J = 30, n = 30	0.5	0.15	Level-2 Slope Var.	5.000	5.323	6.451
J = 30, n = 30	0.5	0.15	Residual Var.	32.500	32.533	0.100
J = 30, n = 30	0.1	0.25	Intercept	49.836	49.813	-0.046
J = 30, n = 30	0.1	0.25	Level-1 Slope	3.098	3.116	0.579
J = 30, n = 30	0.1	0.25	Level-2 Slope	0.724	0.695	-4.040
J = 30, n = 30	0.1	0.25	Level-3 Slope	0.654	0.565	-13.637
J = 30, n = 30	0.1	0.25	Interaction Slope	1.549	1.347	-13.034
J = 30, n = 30	0.1	0.25	Level-3 Intercept Var.	5.104	4.630	-9.280
J = 30, n = 30	0.1	0.25	Level-3 Icept-Slope Cov.	1.917	1.870	-2.436

J = 30, n = 30	0.1	0.25	Level-3 Slope Var.	8.000	7.383	-7.716
J = 30, n = 30	0.1	0.25	Level-2 Intercept Var.	5.500	5.482	-0.327
J = 30, n = 30	0.1	0.25	Level-2 Icept-Slope Cov.	1.990	1.974	-0.793
J = 30, n = 30	0.1	0.25	Level-2 Slope Var.	8.000	8.055	0.692
J = 30, n = 30	0.1	0.25	Residual Var.	52.000	52.030	0.057
J = 30, n = 30	0.5	0.25	Intercept	49.740	49.759	0.038
J = 30, n = 30	0.5	0.25	Level-1 Slope	2.449	2.378	-2.926
J = 30, n = 30	0.5	0.25	Level-2 Slope	1.145	1.140	-0.395
J = 30, n = 30	0.5	0.25	Level-3 Slope	0.938	0.798	-14.907
J = 30, n = 30	0.5	0.25	Interaction Slope	1.225	1.065	-13.029
J = 30, n = 30	0.5	0.25	Level-3 Intercept Var.	11.183	10.593	-5.269
J = 30, n = 30	0.5	0.25	Level-3 Icept-Slope Cov.	2.243	2.175	-3.044
J = 30, n = 30	0.5	0.25	Level-3 Slope Var.	5.000	4.524	-9.525
J = 30, n = 30	0.5	0.25	Level-2 Intercept Var.	13.750	14.683	6.786
J = 30, n = 30	0.5	0.25	Level-2 Icept-Slope Cov.	2.487	1.777	-28.552
J = 30, n = 30	0.5	0.25	Level-2 Slope Var.	5.000	5.519	10.390
J = 30, n = 30	0.5	0.25	Residual Var.	32.500	32.659	0.490
J = 100, n = 30	0.1	0.15	Intercept	49.836	49.841	0.010
J = 100, n = 30	0.1	0.15	Level-1 Slope	3.098	3.092	-0.203
J = 100, n = 30	0.1	0.15	Level-2 Slope	0.724	0.717	-0.999
J = 100, n = 30	0.1	0.15	Level-3 Slope	0.654	0.615	-6.022
J = 100, n = 30	0.1	0.15	Interaction Slope	1.549	1.527	-1.459
J = 100, n = 30	0.1	0.15	Level-3 Intercept Var.	5.104	4.977	-2.487
J = 100, n = 30	0.1	0.15	Level-3 Icept-Slope Cov.	1.917	1.899	-0.914
J = 100, n = 30	0.1	0.15	Level-3 Slope Var.	8.000	7.819	-2.265
J = 100, n = 30	0.1	0.15	Level-2 Intercept Var.	5.500	5.486	-0.246
J = 100, n = 30	0.1	0.15	Level-2 Icept-Slope Cov.	1.990	1.957	-1.652
J = 100, n = 30	0.1	0.15	Level-2 Slope Var.	8.000	8.031	0.384
J = 100, n = 30	0.1	0.15	Residual Var.	52.000	51.981	-0.037
J = 100, n = 30	0.5	0.15	Intercept	49.740	49.778	0.076
J = 100, n = 30	0.5	0.15	Level-1 Slope	2.449	2.426	-0.939
J = 100, n = 30	0.5	0.15	Level-2 Slope	1.145	1.178	2.866
J = 100, n = 30	0.5	0.15	Level-3 Slope	0.938	0.907	-3.370
J = 100, n = 30	0.5	0.15	Interaction Slope	1.225	1.168	-4.645
J = 100, n = 30	0.5	0.15	Level-3 Intercept Var.	11.183	11.219	0.324
J = 100, n = 30	0.5	0.15	Level-3 Icept-Slope Cov.	2.243	2.189	-2.408
J = 100, n = 30	0.5	0.15	Level-3 Slope Var.	5.000	4.759	-4.829
J = 100, n = 30	0.5	0.15	Level-2 Intercept Var.	13.750	14.419	4.869
J = 100, n = 30	0.5	0.15	Level-2 Icept-Slope Cov.	2.487	2.029	-18.428
J = 100, n = 30	0.5	0.15	Level-2 Slope Var.	5.000	5.295	5.904
J = 100, n = 30	0.5	0.15	Residual Var.	32.500	32.578	0.239
J = 100, n = 30	0.1	0.25	Intercept	49.836	49.838	0.004
J = 100, n = 30	0.1	0.25	Level-1 Slope	3.098	3.109	0.357

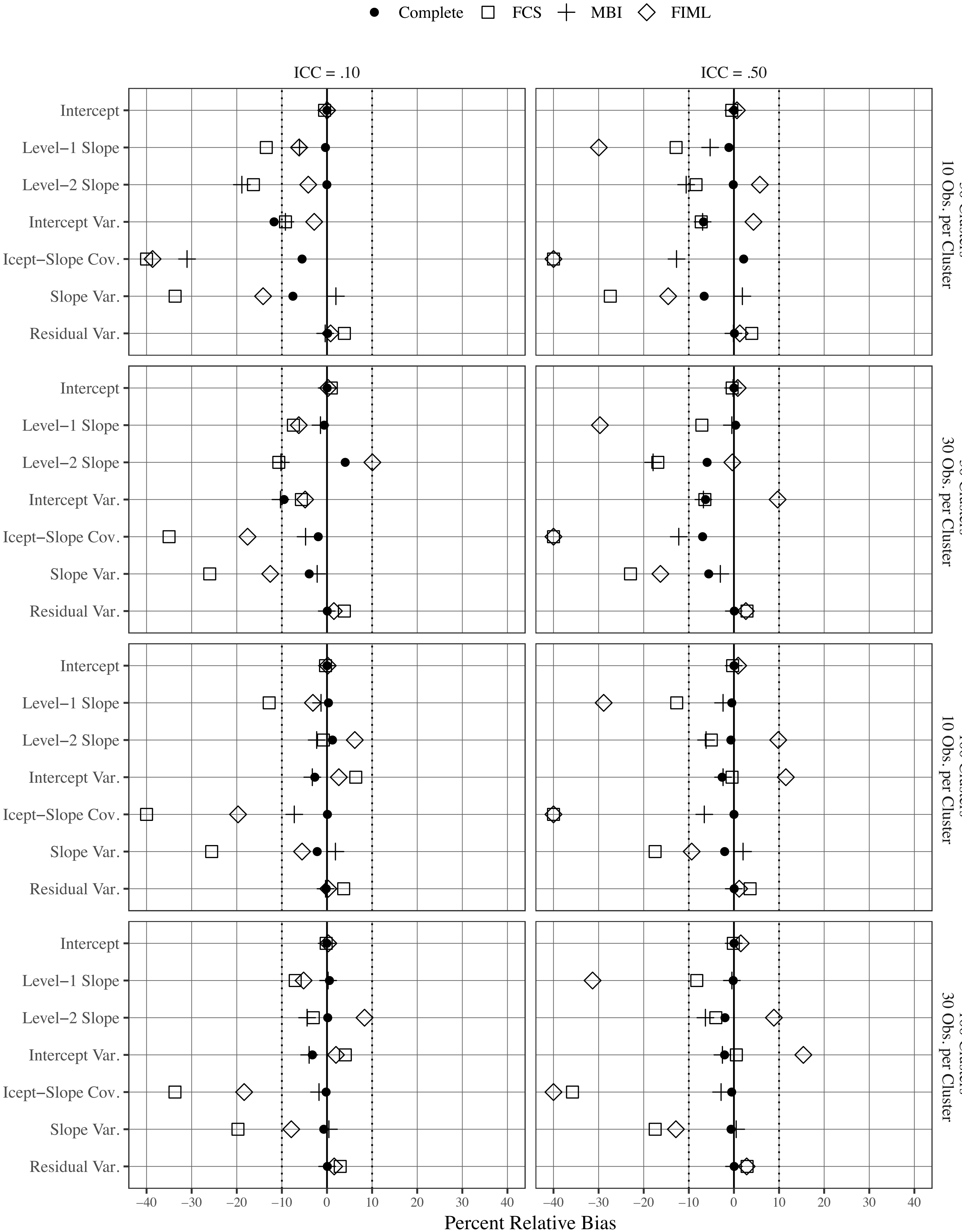
J = 100, n = 30	0.1	0.25	Level-2 Slope	0.724	0.702	-3.018
J = 100, n = 30	0.1	0.25	Level-3 Slope	0.654	0.635	-2.960
J = 100, n = 30	0.1	0.25	Interaction Slope	1.549	1.483	-4.295
J = 100, n = 30	0.1	0.25	Level-3 Intercept Var.	5.104	4.967	-2.691
J = 100, n = 30	0.1	0.25	Level-3 Icept-Slope Cov.	1.917	1.929	0.604
J = 100, n = 30	0.1	0.25	Level-3 Slope Var.	8.000	7.861	-1.739
J = 100, n = 30	0.1	0.25	Level-2 Intercept Var.	5.500	5.517	0.305
J = 100, n = 30	0.1	0.25	Level-2 Icept-Slope Cov.	1.990	1.937	-2.675
J = 100, n = 30	0.1	0.25	Level-2 Slope Var.	8.000	8.098	1.227
J = 100, n = 30	0.1	0.25	Residual Var.	52.000	52.046	0.089
J = 100, n = 30	0.5	0.25	Intercept	49.740	49.784	0.088
J = 100, n = 30	0.5	0.25	Level-1 Slope	2.449	2.396	-2.200
J = 100, n = 30	0.5	0.25	Level-2 Slope	1.145	1.150	0.444
J = 100, n = 30	0.5	0.25	Level-3 Slope	0.938	0.896	-4.458
J = 100, n = 30	0.5	0.25	Interaction Slope	1.225	1.138	-7.122
J = 100, n = 30	0.5	0.25	Level-3 Intercept Var.	11.183	11.405	1.994
J = 100, n = 30	0.5	0.25	Level-3 Icept-Slope Cov.	2.243	2.156	-3.879
J = 100, n = 30	0.5	0.25	Level-3 Slope Var.	5.000	4.737	-5.261
J = 100, n = 30	0.5	0.25	Level-2 Intercept Var.	13.750	14.586	6.077
J = 100, n = 30	0.5	0.25	Level-2 Icept-Slope Cov.	2.487	1.833	-26.306
J = 100, n = 30	0.5	0.25	Level-2 Slope Var.	5.000	5.555	11.091
J = 100, n = 30	0.5	0.25	Residual Var.	32.500	32.598	0.303

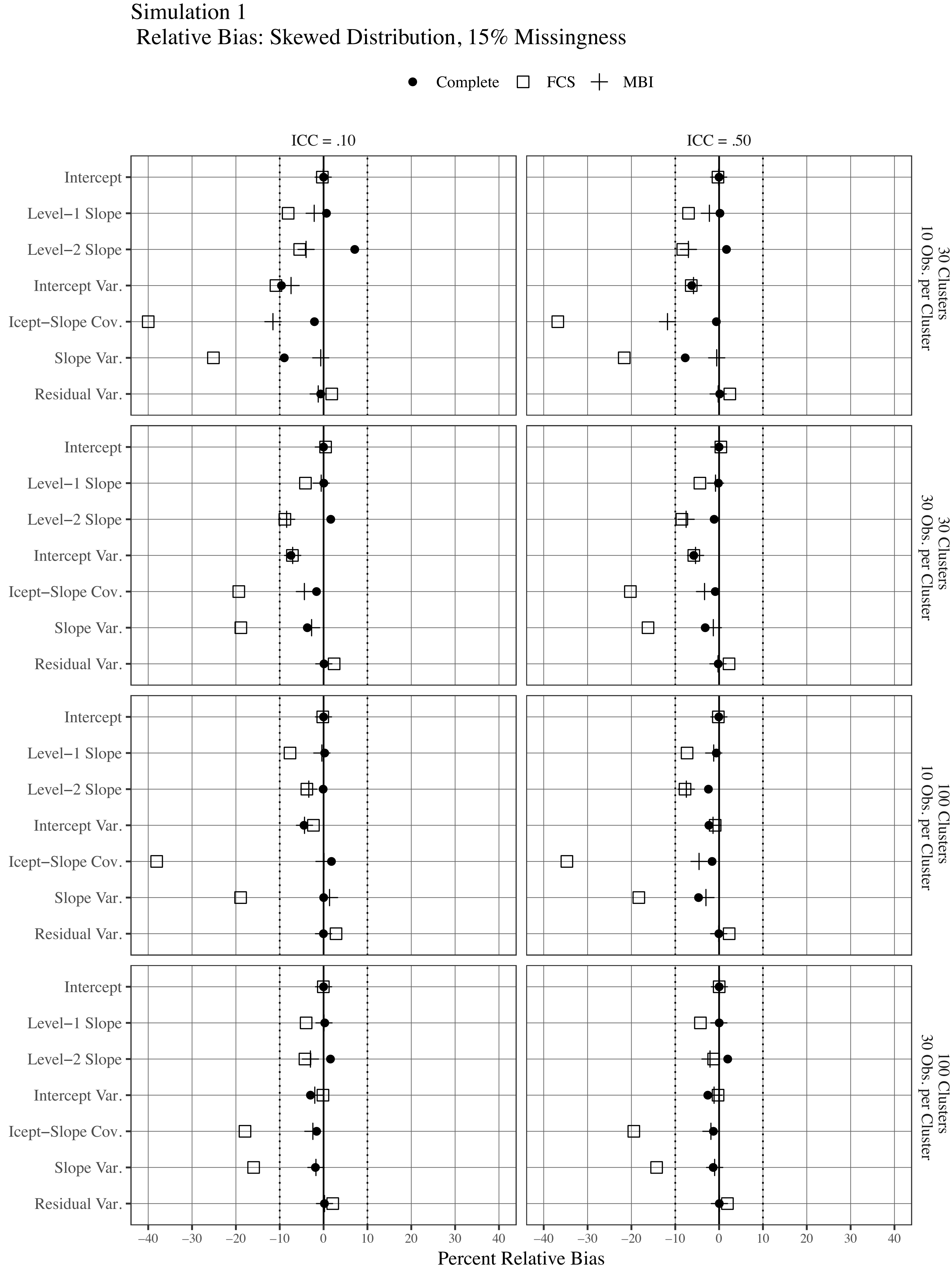

Simulation 1: Relative Bias, Large Sample (N = 50,000)
Complete
FCS
MBI
LWD
ICC = .10
ICC = .50
Intercept
Level−1 Slope
Level−2 Slope
Intercept Var.
Icept−Slope Cov.
Slope Var.
Residual Var.
Normal Level−2 Covariate 15% Missingness
Normal Level−2 Covariate 25% Missingness
Skewed Level−2 Covariate 15% Missingness
Skewed Level−2 Covariate 25% Missingness
−40
−30
−20
−10
0
10
20
30
40
Percent Relative Bias

Simulation 1
Relative Bias: Normal Distribution, 15% Missingness

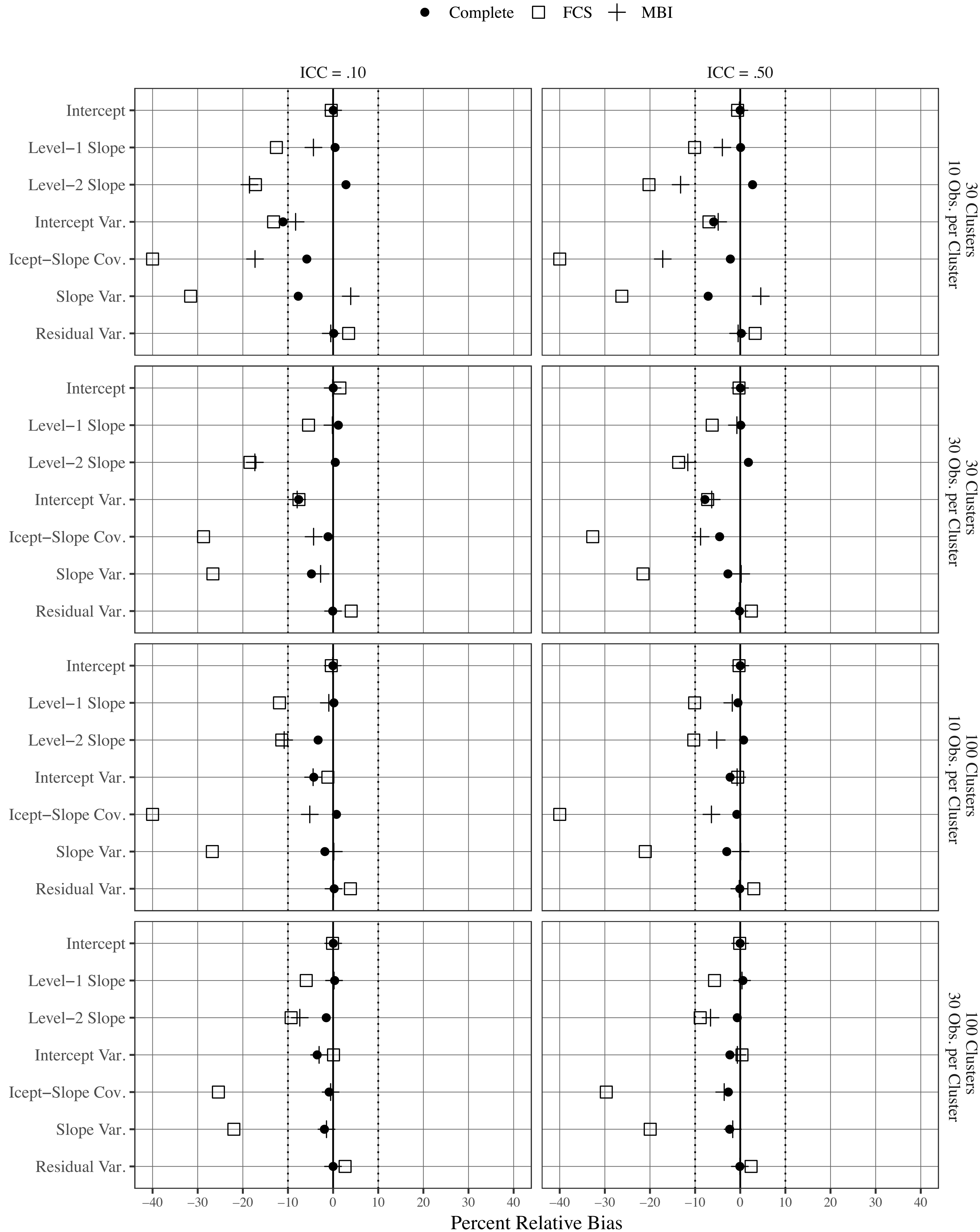


Simulation 1
Relative Bias: Normal Distribution, 25% Missingness

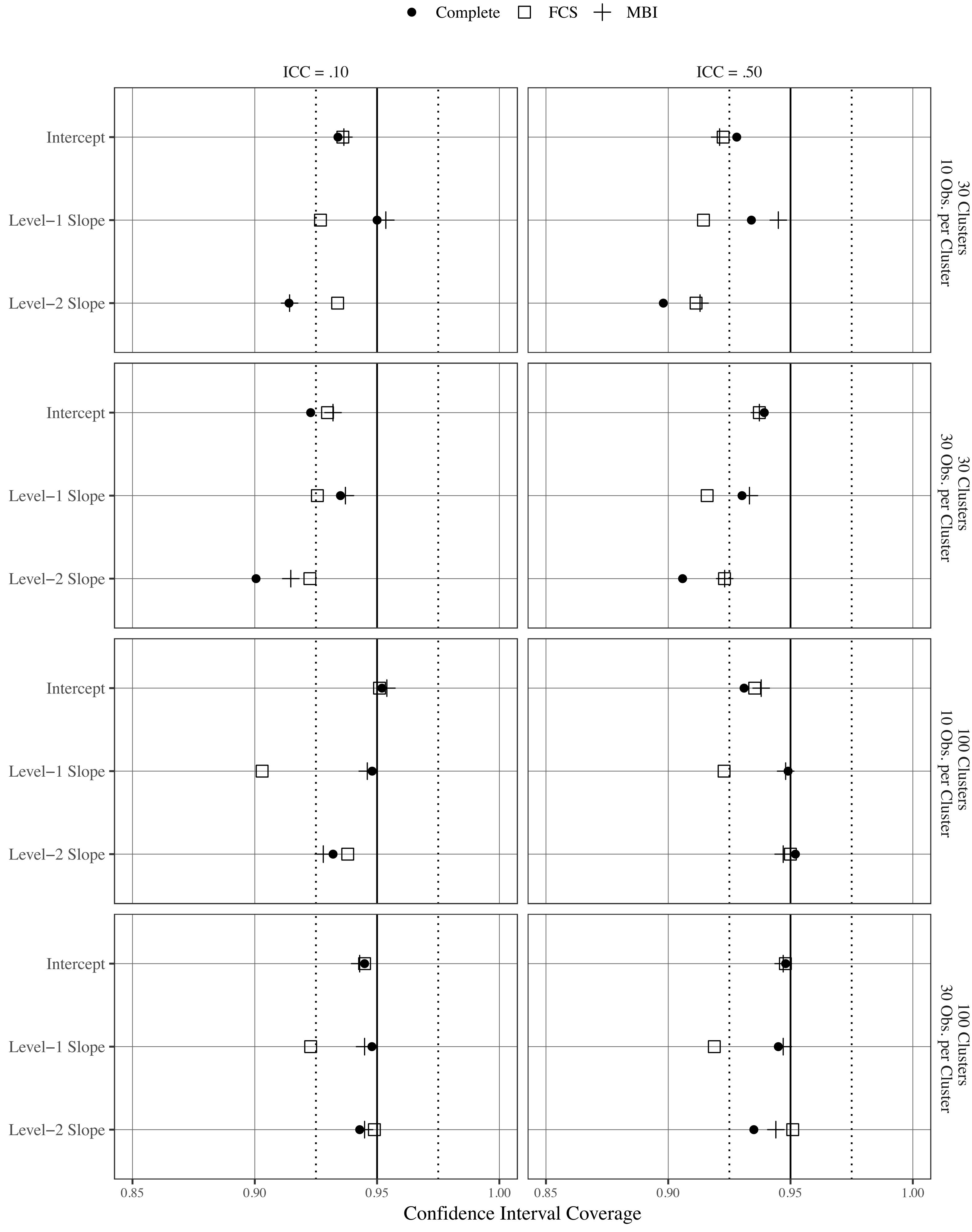



Simulation 1
Relative Bias: Skewed Distribution, 15% Missingness
Complete
FCS
MBI
ICC = .10
ICC = .50
Intercept
Level−1 Slope
Level−2 Slope
Intercept Var.
Icept−Slope Cov.
Slope Var.
Residual Var.
30 Clusters 10 Obs. per Cluster
30 Clusters 30 Obs. per Cluster
100 Clusters 10 Obs. per Cluster
100 Clusters 30 Obs. per Cluster
−40 −30 −20 −10 0 10 20 30 40
Percent Relative Bias

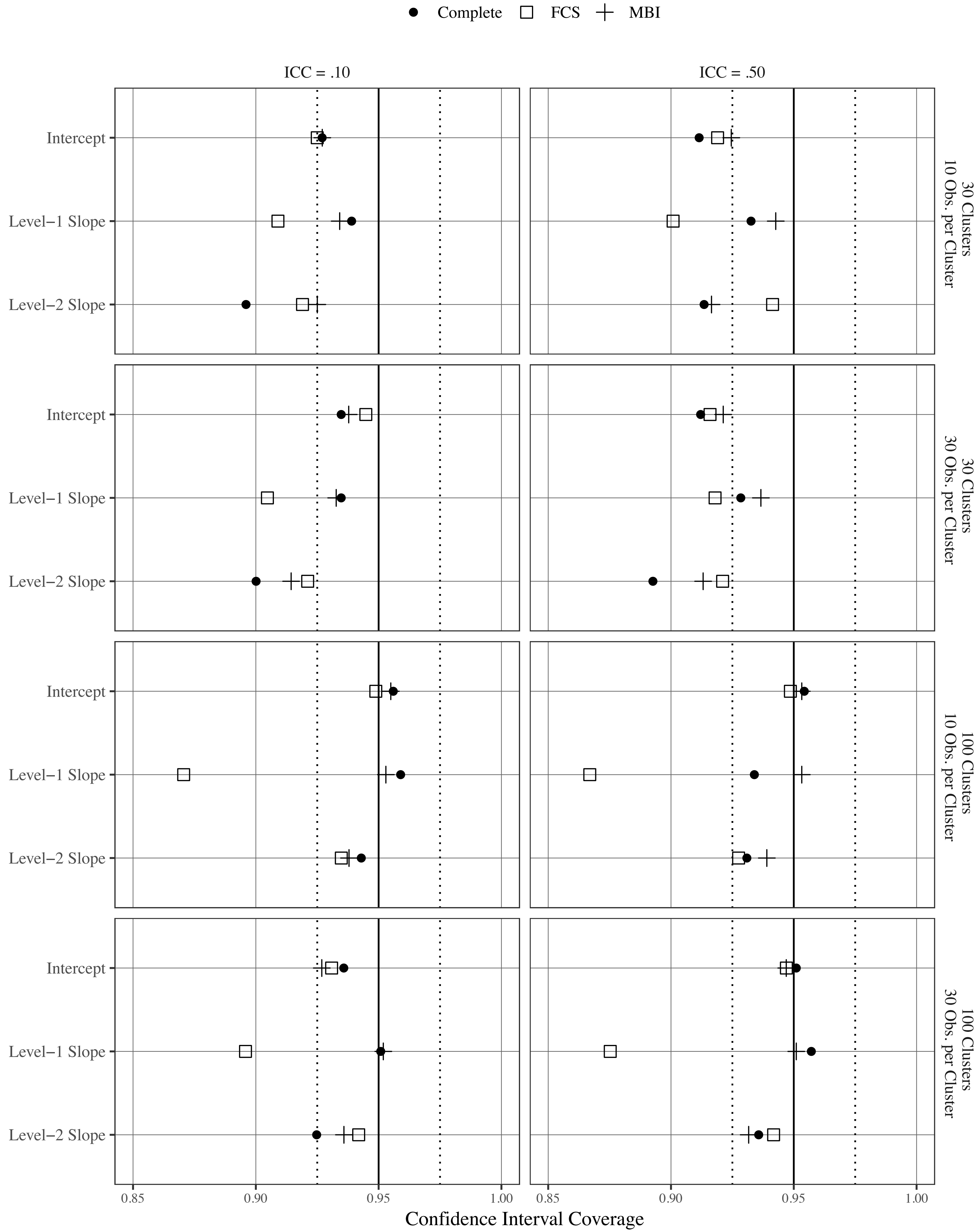
Simulation 1
Relative Bias: Skewed Distribution, 25% Missingness



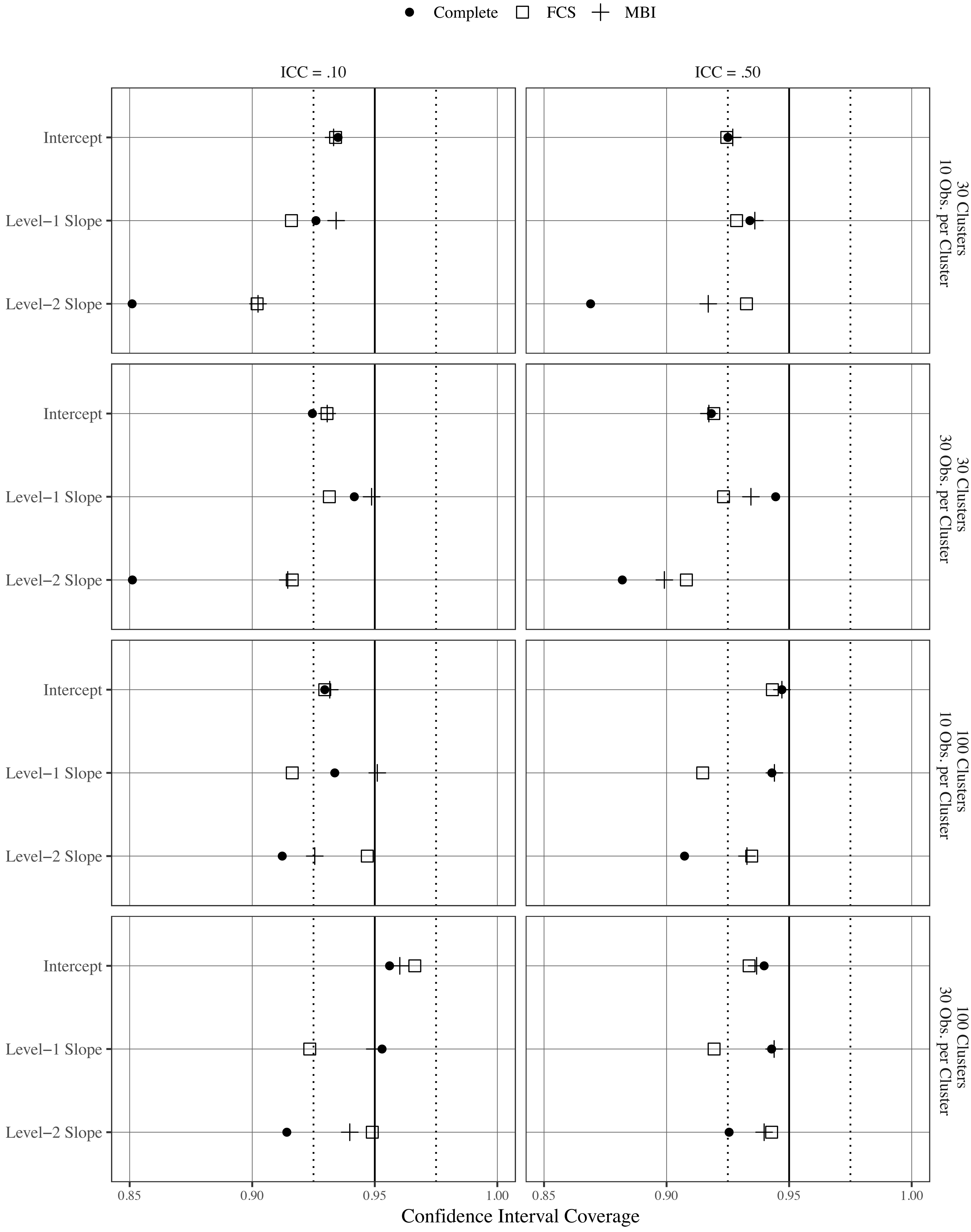
Simulation 1
Interval Coverage: Normal Distribution, 15% Missingness



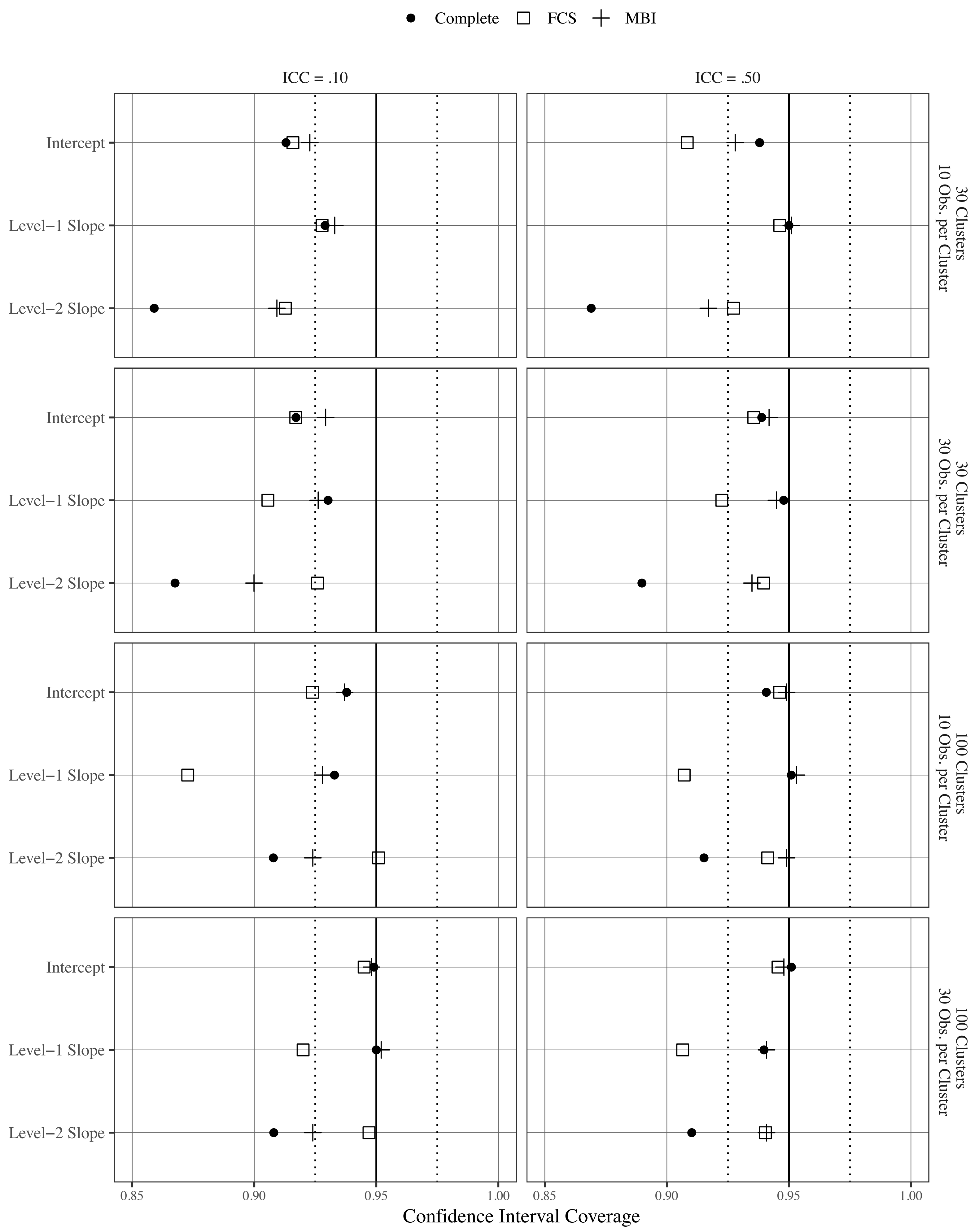
Simulation 1
Interval Coverage: Normal Distribution, 25% Missingness

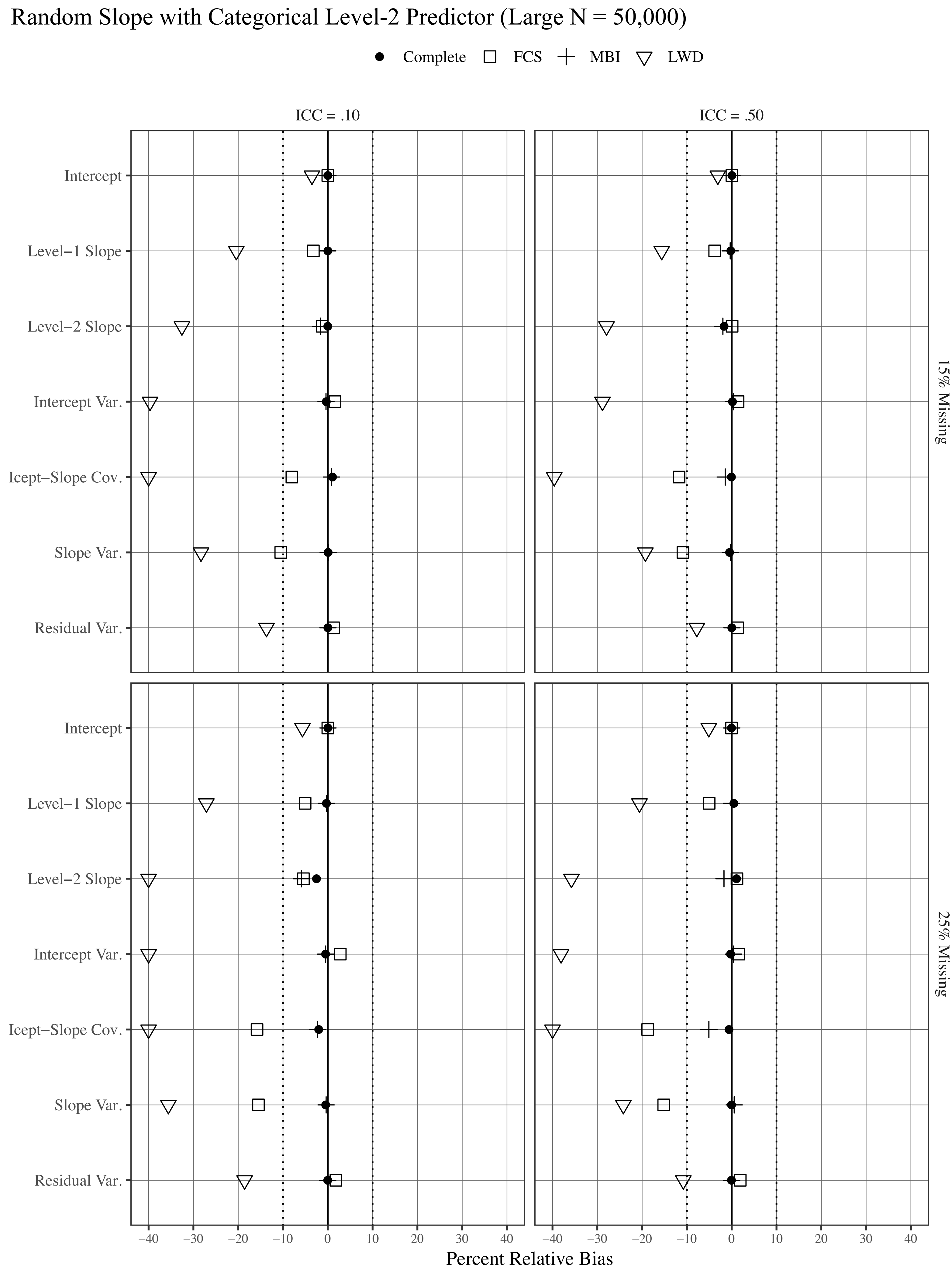


Simulation 1
Interval Coverage: Skewed Distribution, 15% Missingness



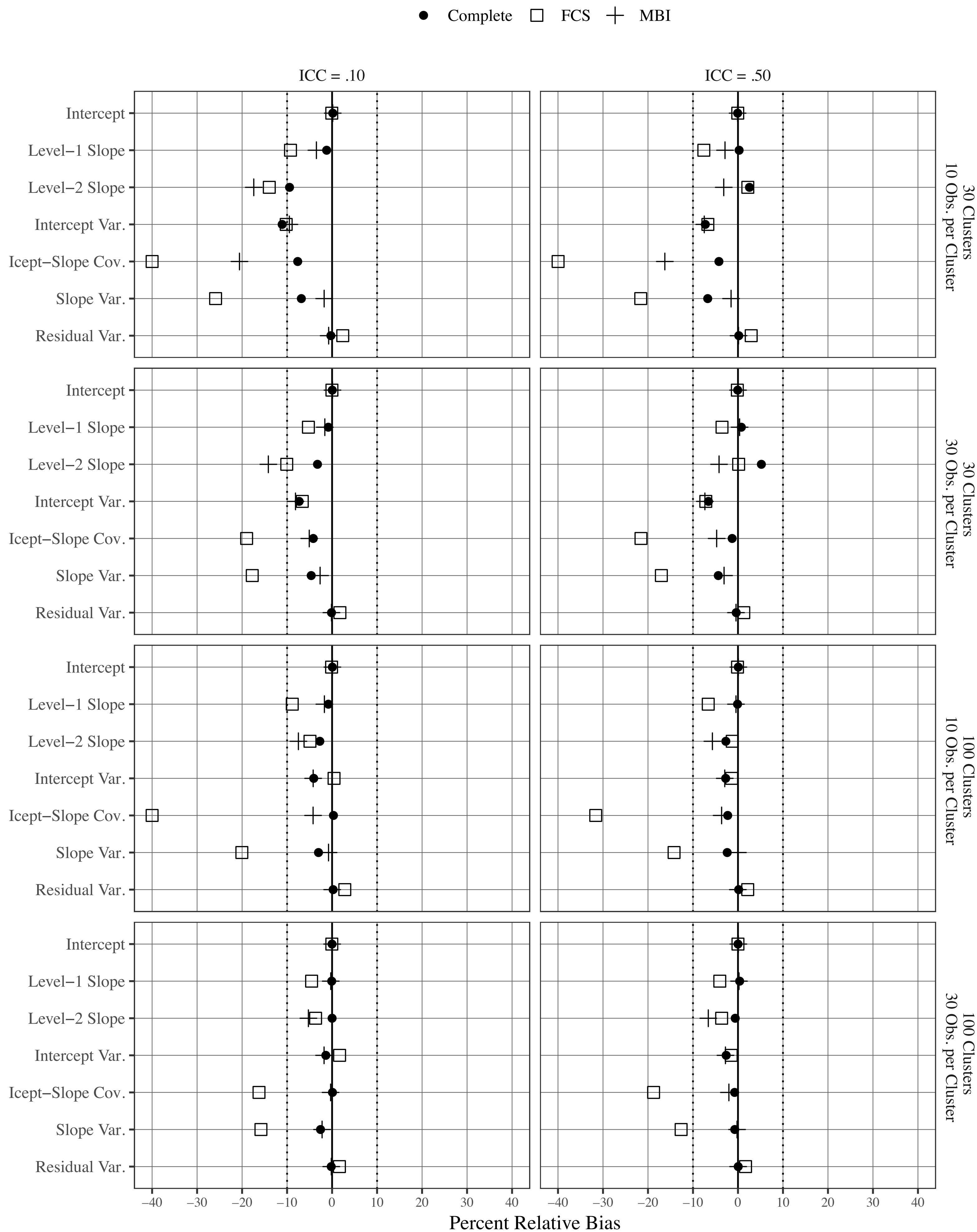
Simulation 1
Interval Coverage: Skewed Distribution, 25% Missingness



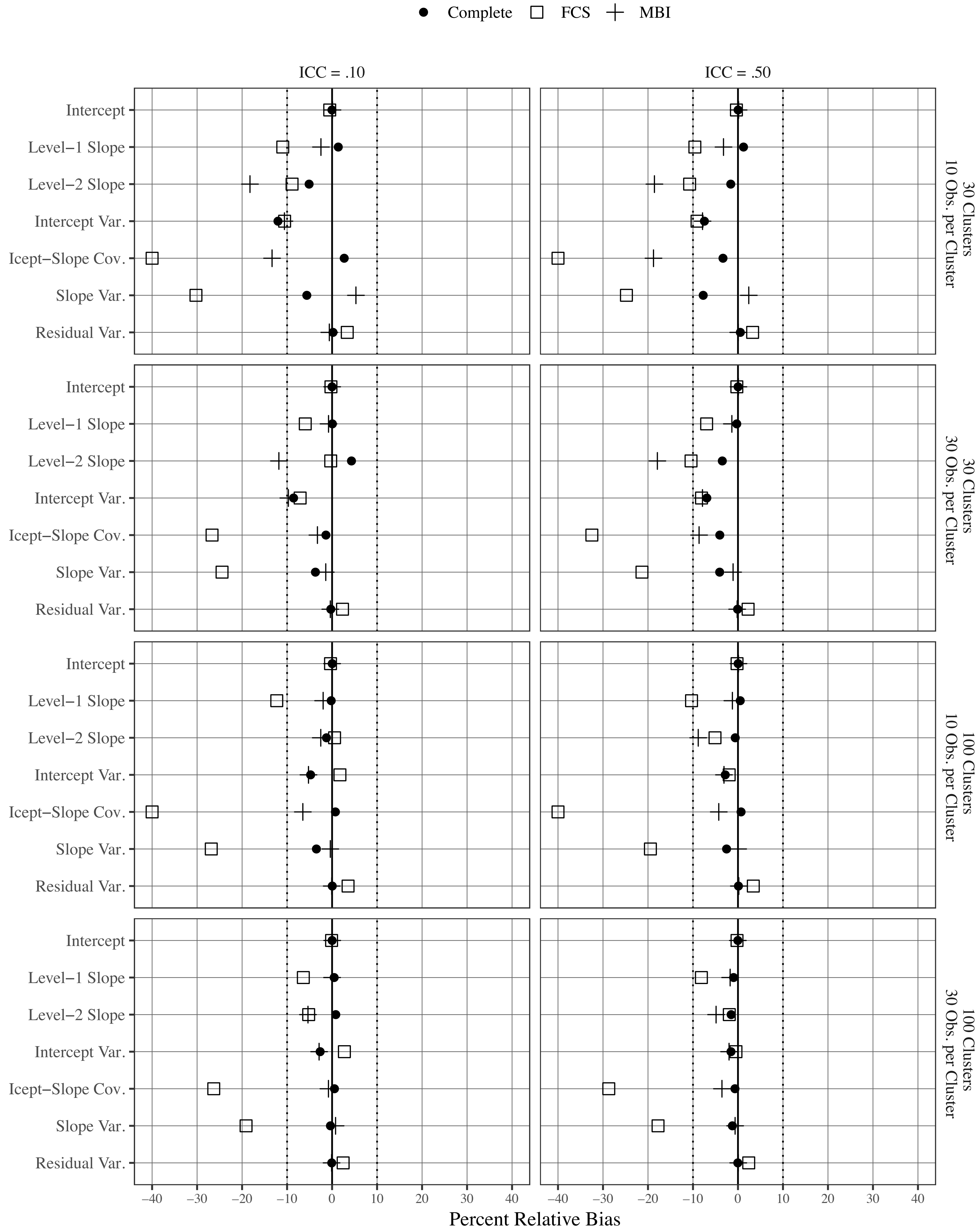


Random Slope with Categorical Level-2 Predictor (Large N = 50,000)
Complete
FCS
MBI
LWD
ICC = .10
ICC = .50
Intercept
Level−1 Slope
Level−2 Slope
Intercept Var.
Icept−Slope Cov.
Slope Var.
Residual Var.
15% Missing
25% Missing
−40 −30 −20 −10 0 10 20 30 40
Percent Relative Bias

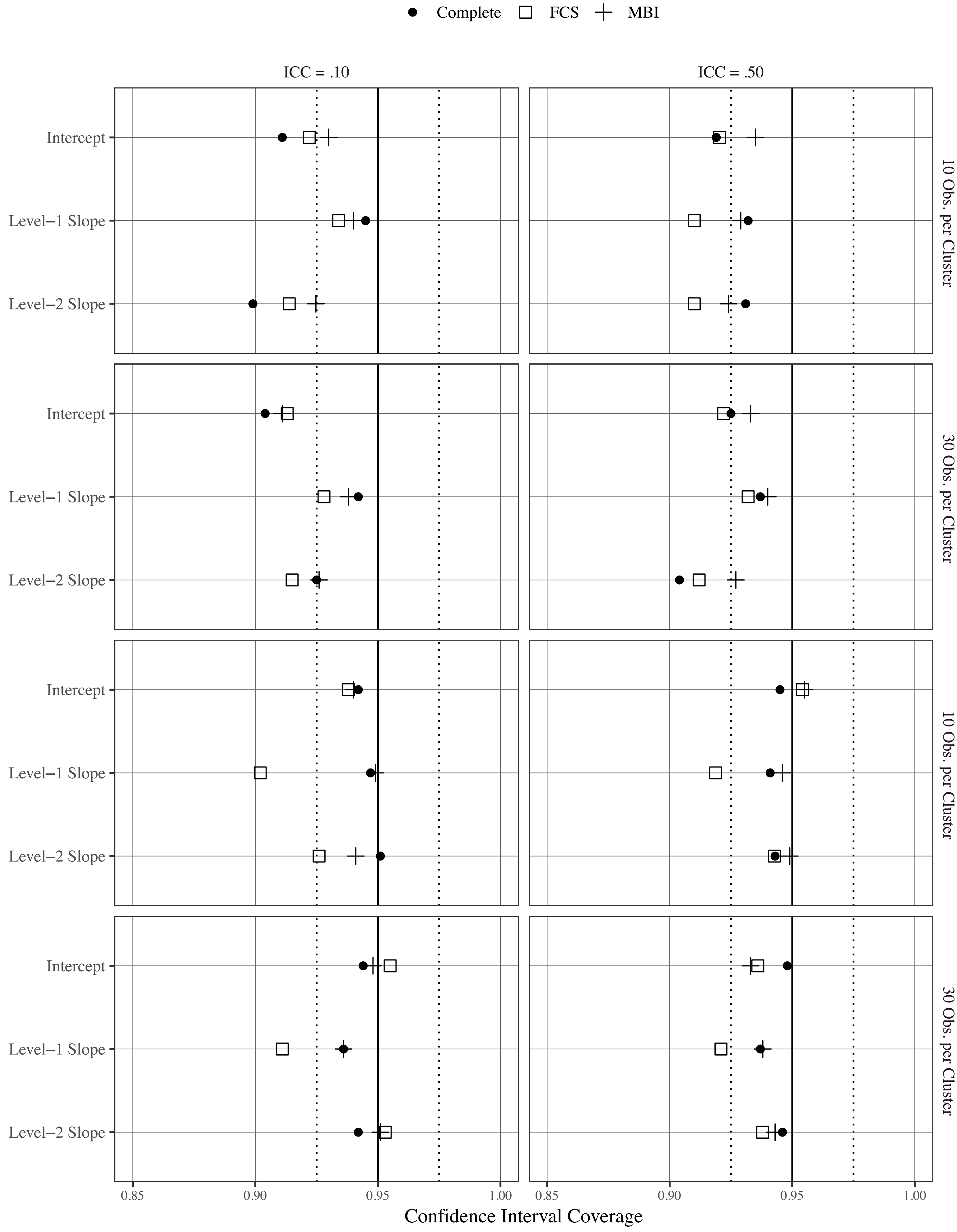
Simulation 2
Relative Bias: 15% Missingness



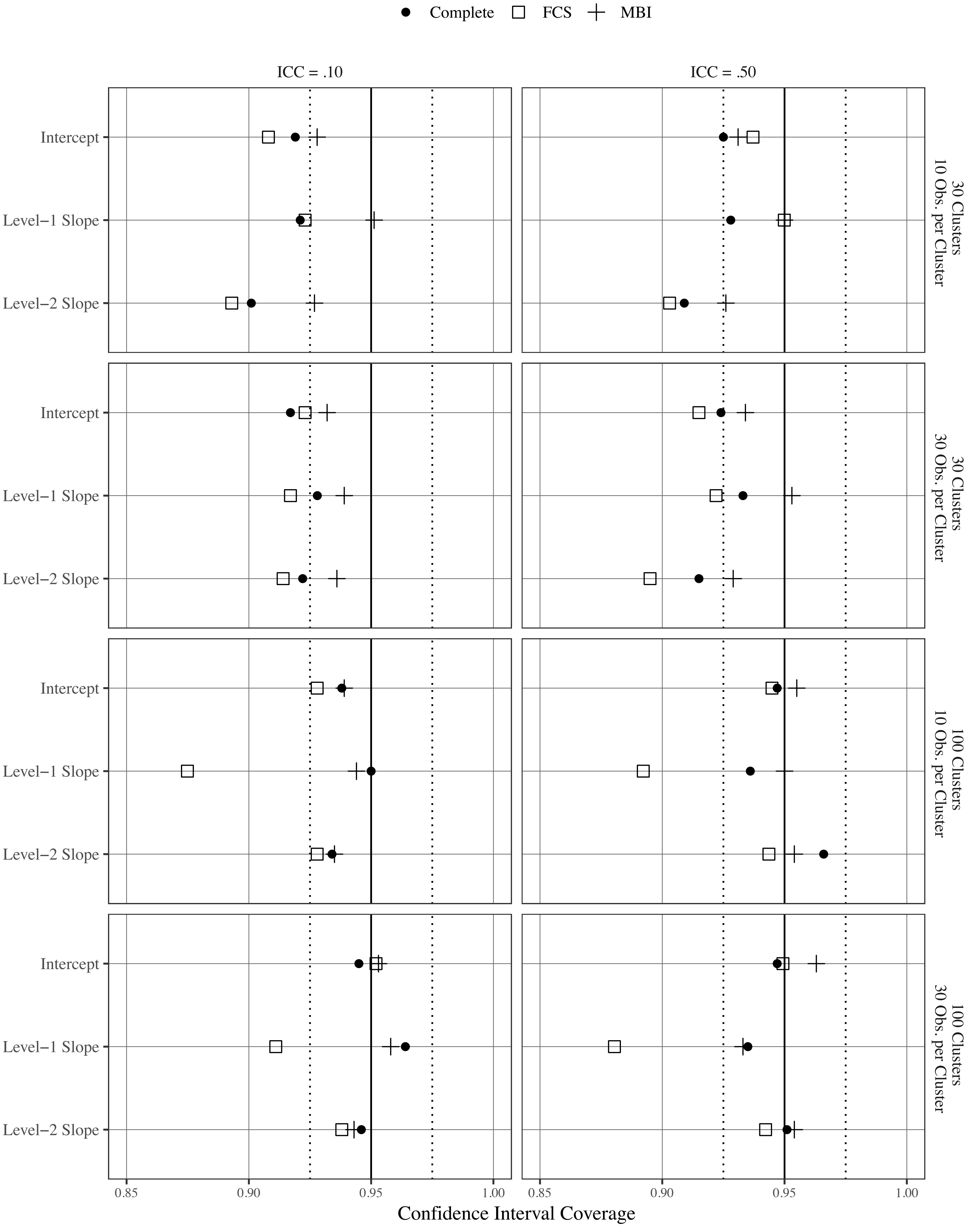
Simulation 2
Relative Bias: 25% Missingness



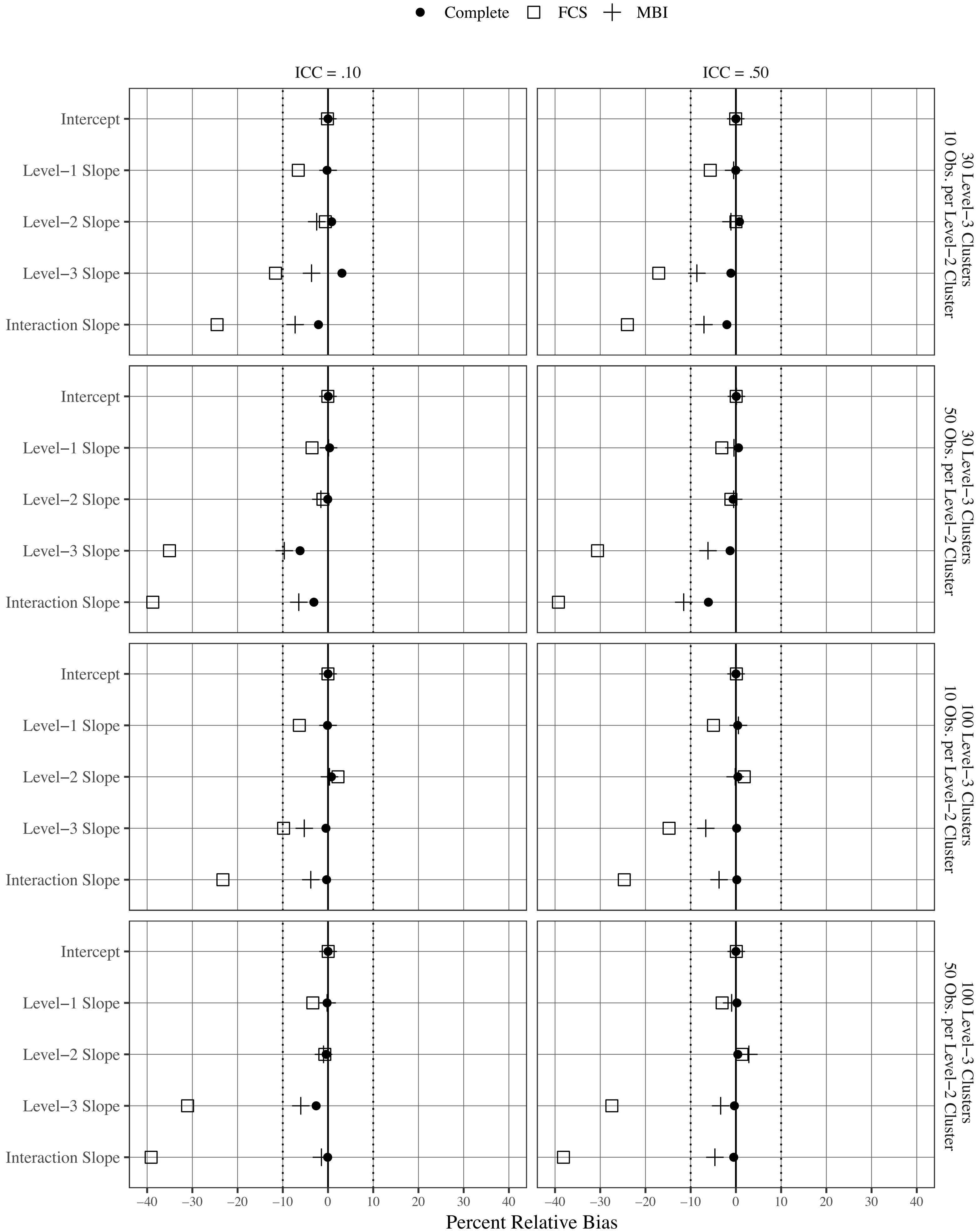
Simulation 2
Interval Coverage: 15% Missingness



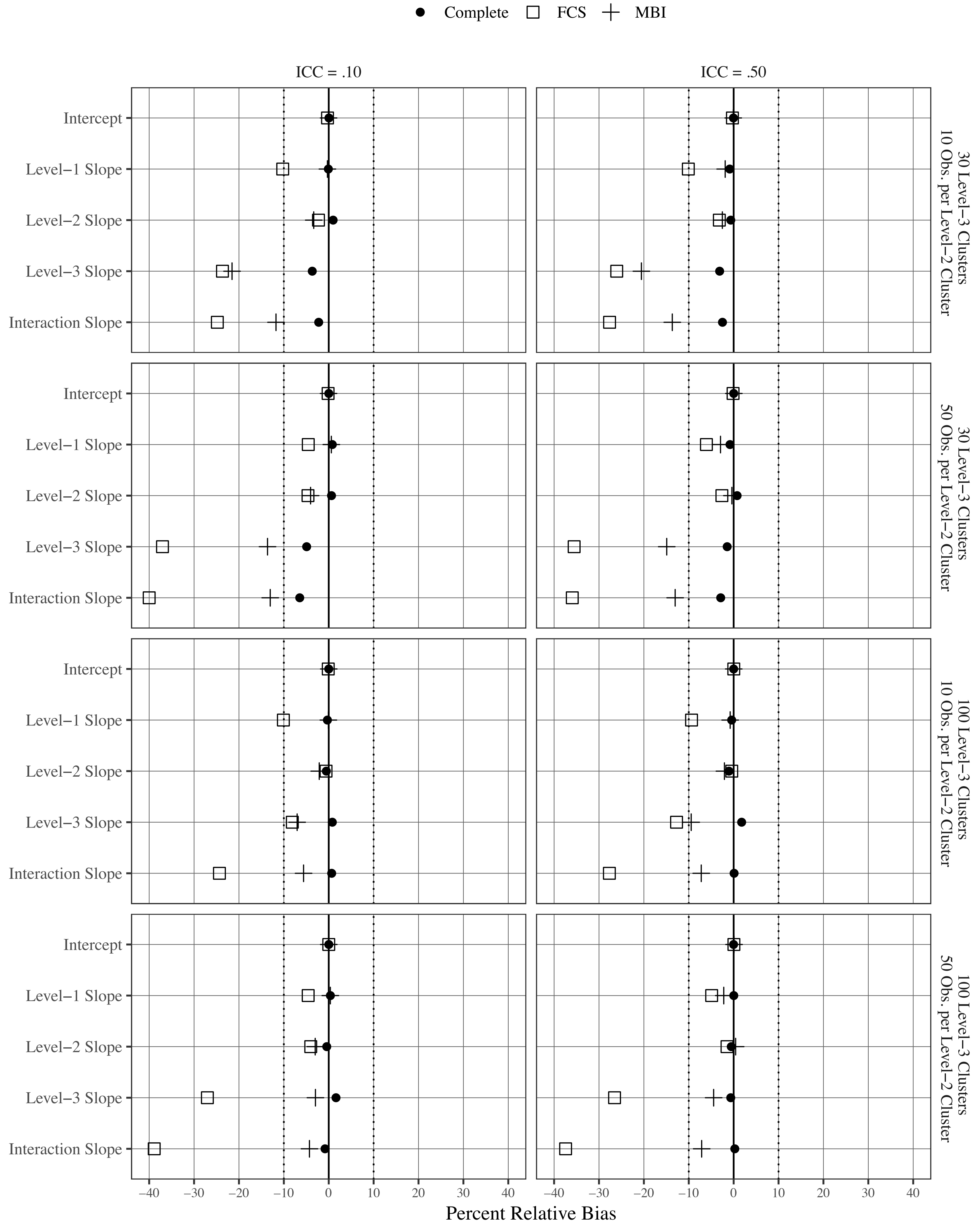
Simulation 2
Interval Coverage: 25% Missingness



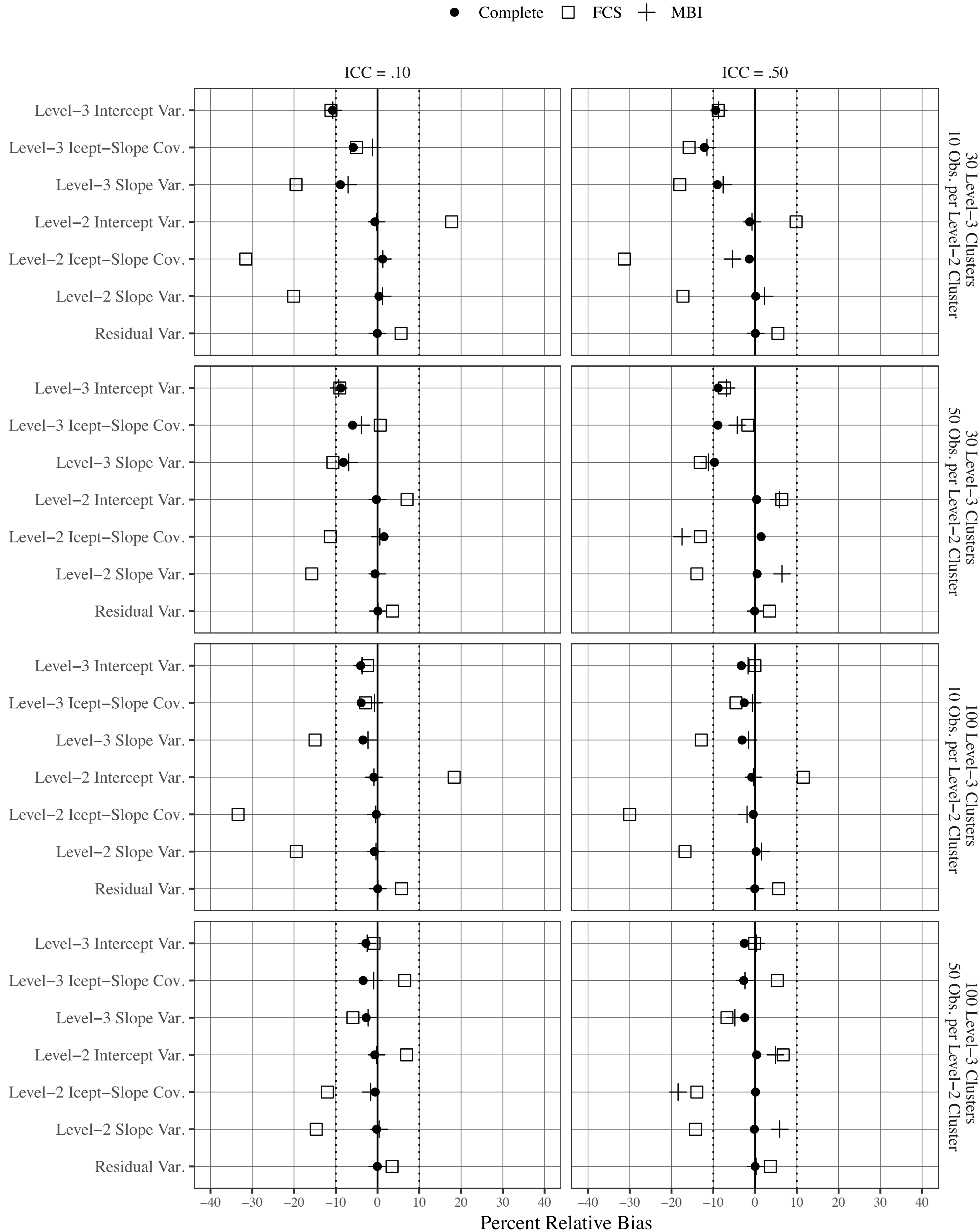
Simulation 3
Relative Bias: 15% Missingness



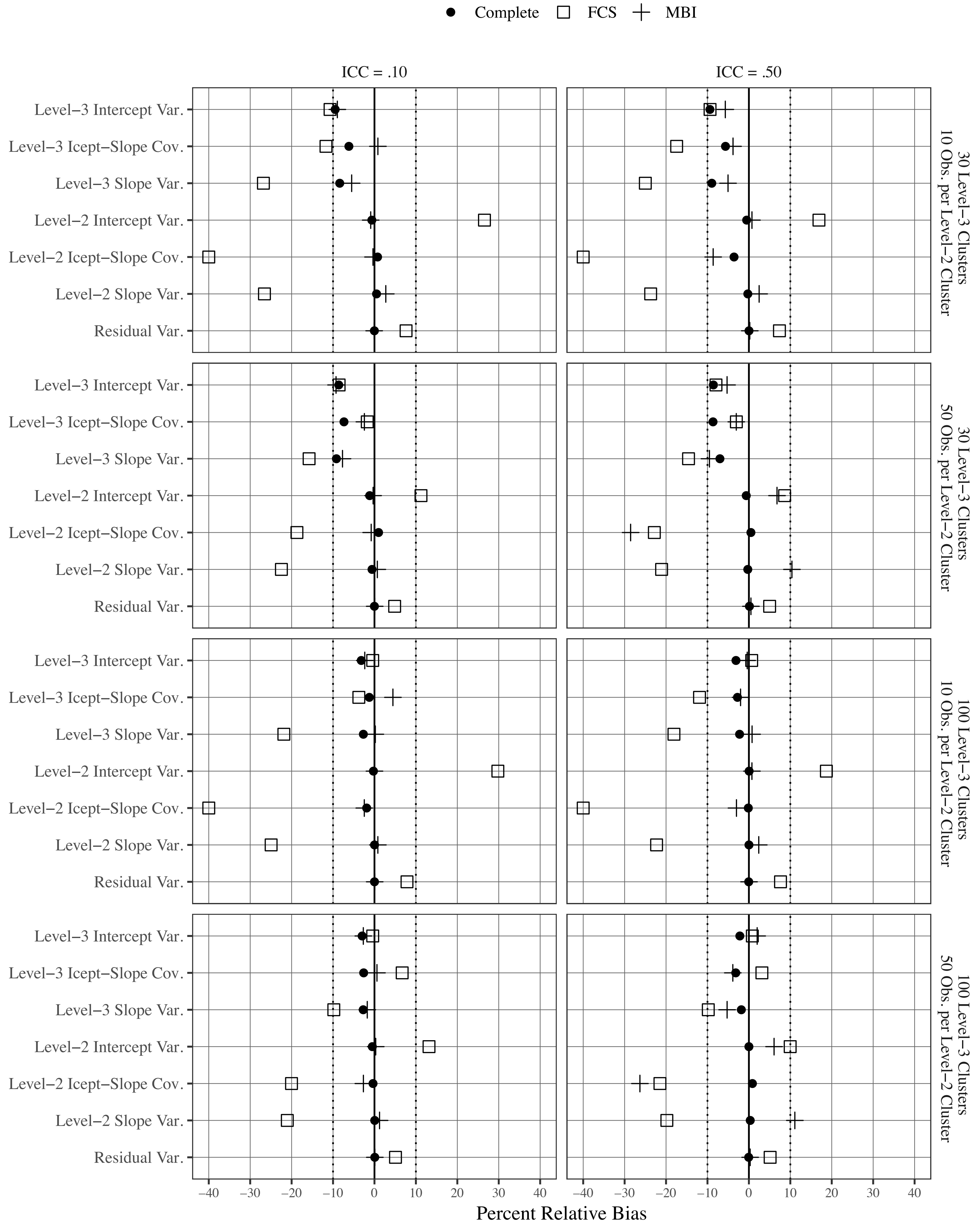
Simulation 3
Relative Bias: 25% Missingness



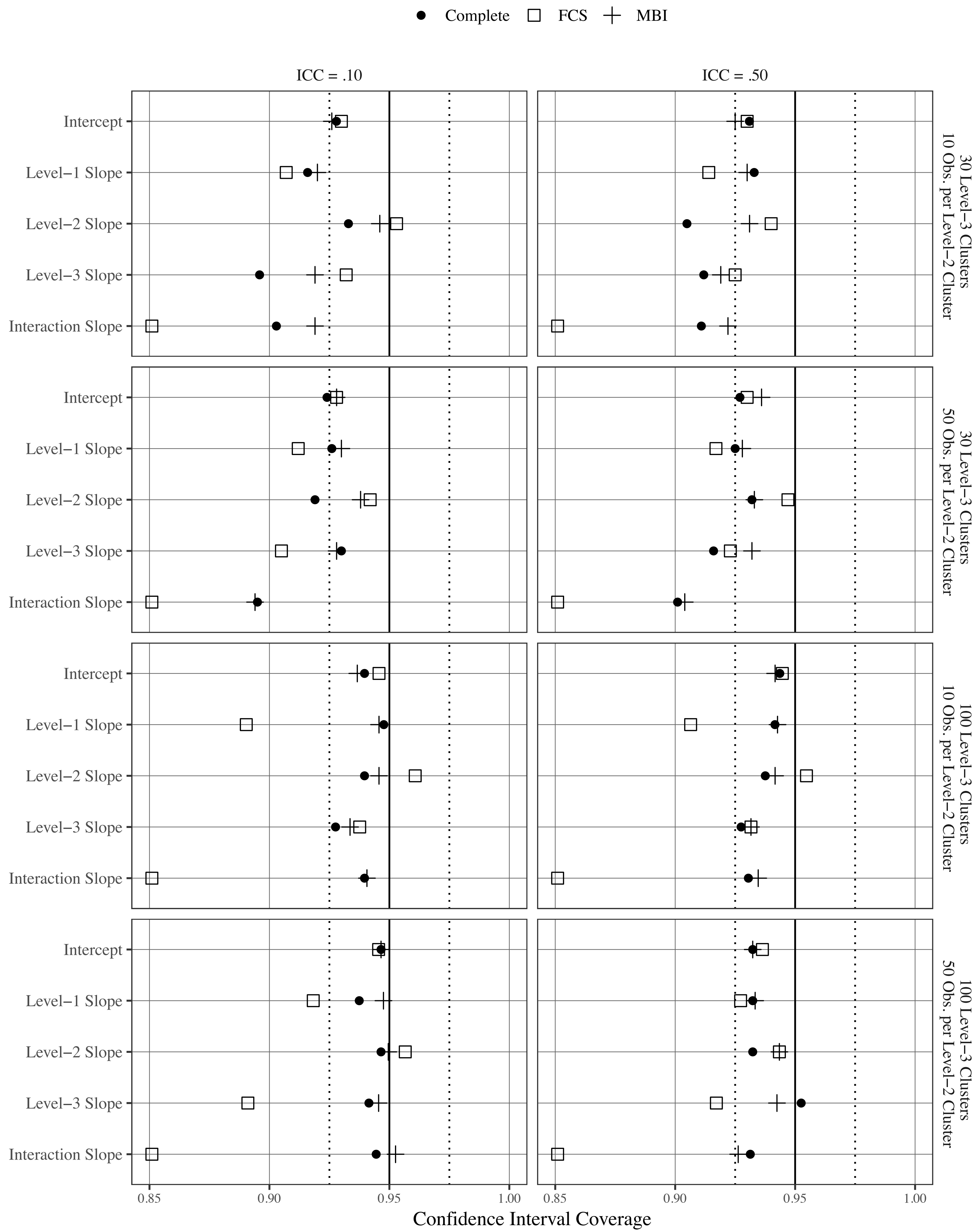
Simulation 3
Relative Bias: 15% Missingness



Simulation 3
Relative Bias: 25% Missingness



Simulation 3
Interval Coverage: 15% Missingness



Simulation 3
Interval Coverage: 25% Missingness